Compressibility effects in free shear layers

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High Reynolds number compressible free shear layers were studied experimentally to explore the effects of compressibility on the turbulence field. Previous preliminary results reported by the authors showed that the level and the lateral extent of turbulence fluctuations are reduced as the compressibility, which is characterized by a convective Mach number, is increased. The two convective Mach numbers used in the previous study were relatively close, $M_c = 0.51$ and 0.64, and as a result the conclusions were not concrete. The present results with $M_c = 0.86$ strongly support the earlier results, showing even higher reductions in the level and the lateral extent of Reynolds stresses. The higher-order moments of turbulence fluctuations such as skewness and flatness are reported, which show that the intermittency resulting from the excursions of large-scale structures into the free stream at the edge of shear layers was significantly reduced (both in the level and the extent) because of increased $M_c$. In the developing region of shear layers, development of mean flow and turbulence fluctuation profiles are reported that have similar trends seen in incompressible shear layers.

I. INTRODUCTION

A recent revival of supersonic combustion and propulsion research has been initiated by the renewed interest in an air-breathing hypersonic vehicle. The SCRAMJET (supersonic combustion ramjet) has been proposed as a possible engine to mix and combust the fuel and oxidizer at supersonic speeds. There are many possible configurations of the SCRAMJET, common in all of these proposed configurations is the need to understand how a fuel-rich flow will mix and combust with a supersonic oxidizer flow. A simplified cold flow model of a two-stream free shear layer has been examined experimentally in order to obtain some insight into the mixing processes in such a system. Figure 1 gives a schematic of the two-stream flow field. This free shear layer model has been previously studied by the authors and further characteristics of it will be discussed in this paper.

The original notion that the density gradient across free shear layers was the cause of significant growth rate decrease was disputed by detailed experimental investigations of an equivalent incompressible free shear layer conducted by Brown and Roshko. Recent experimental results by Pappaschou and Roshko and theoretical results by Bogdanoff showed that compressibility is the main cause of lower growth rates in compressible shear layers. The convective Mach number, a Mach number with respect to a frame of reference traveling with the average large-scale structures in the flow, has been identified as a compressibility parameter. The convective Mach number seems to correlate the effect of compressibility on the shear layer growth rate normalized by its incompressible counterpart. If the two streams of a compressible free shear layer have the same specific heats ratio, the convective Mach numbers of the two streams become the same and can be given by

$$M_c = \frac{(U_1 - U_2)}{a_1},$$

where $U_c$ is the convective velocity of the large-scale structures propagating with the flow given by

$$U_c = a_1 U_2 + a_2 U_1 / (a_1 + a_2),$$

where $U_1$, $U_2$ and $a_1$, $a_2$ are the free-stream velocities and speeds of sound of the upper and lower streams, respectively. Although the convective Mach number gives a parameter for the effect of compressibility on the growth rate of free shear layers, its ability to correlate the effects of compressibility on other free shear layer characteristics is still open to further research.

Incompressible planar free shear layers have undergone intense research activity in the past with a substantial amount of material published on the mean flow quantities, turbulence properties, flow visualization, and development to self-similarity of the flow field. Recently, there has also been a great deal of experimental and computational work in the area of compressible free shear layers. Most of the recent experimental work on compressible free shear layers has been in the areas of mean flow measurements and the effect of compressibility on the shear layer growth rate. Samimi and Elliott showed that not only is the growth rate decreasing with increasing convective Mach number, but the turbulence quantities are also decreasing.

There have been many works on the stability of compressible shear layers. In the context of convective Mach number, Ragab and Wu looked at the linear spatial instability of compressible laminar mixing of two parallel streams investigating the effects of temperature and velocity ratios, Reynolds number, and convective Mach number. Zhuang, Kubota, and Dimotakis used a similar analysis, except looking only at the inviscid disturbances. They found that there is a nearly universal dependence of the normalized maximum amplification rate of disturbances on the convective Mach number, similar to experimental growth rate results. Using direct numerical simulation, Lele investigated the contribution of each term in the inviscid vorticity equation as the convective Mach number was increased. Sandham and Reynolds extended Lele's simulation to three dimensions and showed that the structures become increasingly three dimensional as the convective Mach number is increased.
The work presented in this paper is part of an ongoing research activity to investigate the effects of compressibility on free shear layers. Previously, work has been reported on the effects of compressibility on mean flow and turbulence characteristics for two relatively close convective Mach numbers. The present research increases the convective Mach number range, substantiates the accuracy of turbulence measurements using LDV, and further investigates the turbulence characteristics of the free shear layer, including the development of the shear layer and higher-order turbulence quantities. The next section provides a brief description of the experimental facility and instrumentation followed by the results and discussions.

II. EXPERIMENTAL FACILITY AND TECHNIQUES

The experiments were conducted at the Ohio State University Aeronautical and Astronomical Research Laboratory (AARL). The newly refurbished high Reynolds number supersonic blow-down tunnel is a dual-stream tunnel with a 152.4 mm by 152.4 mm test section. A 3.175 mm thick steel splitter plate with a machined angle on the subsonic side of approximately 1° over a 125 mm length and a flat profile on the supersonic side, separates the two flows. The trailing edge of the splitter plate has a thickness of about 0.5 mm. Optical access to the side of the test section is provided giving a possible viewing area approximately 80 mm high and 500 mm long. Two separate control valves make it possible to regulate the flow in the top and bottom flows. A more detailed description of the experimental setup has been given previously.

The three convective Mach number cases were obtained by different Mach number combinations in the two streams. Two different convective Mach numbers were 0.51 and 0.64 for cases 1 and 2, respectively. Incoming and mean flow parameters for these two cases have been reported earlier. The pertinent mean flow parameters for the subsonic data from Oster and Wyganski are also summarized in these references. For the third case (case 3), using a nominal Mach 3.0 nozzle on the supersonic side, the total pressure, total temperature, and velocity were 265 kPa, 276 K, and 597 m/sec, respectively. The incoming supersonic fully turbulent incoming boundary layer had a thickness of 9.2 mm and a momentum thickness of 0.46 mm giving a Reynolds number of 24 700 based on the incoming boundary moment thickness. The supersonic stream fully expanded at the edge of the splitter plate, thus generating a very weak expansion or compression wave. To create this shear layer the static pressure was matched on the subsonic side with \( M_s = 0.45 \), giving a velocity ratio of 0.25 and a density ratio of 0.37. The convective Mach number of the mixing layer for this case was 0.86.

A two-component coincident laser Doppler velocimetry system, set at a 10° off-axis forward scattering mode, was used in these experiments. Both flows were seeded in the settling chamber with atomized silicone oil less than 1 µm in diameter. All the LDV results presented here are based on 2048 samples per channel. Although the number of samples taken are sufficient for mean velocity results, a larger sample size would be desirable to reduce the scattering in the turbulence results.

Two schlieren systems were used for flow visualization studies of the free shear layer: one is a standard system with a 500 nsec flash and a 35 mm camera; the other is a laser-based system utilizing an intensified CCD camera to record the images, which are saved on a Panasonic S-VHS video recorder with editing features. In Fig. 2 we present a typical schlieren photograph taken in the self-similar region for case 3 from the laser based CCD system. The schlieren photograph indicates the presence of large-scale structures appearing randomly and somewhat unorganized, similar to those observed in other supersonic free shear layers, supersonic reattaching shear layers, and supersonic boundary layers. The spanwise extent of these structures is lost as a result of the spanwise averaging of the schlieren technique. Work is underway to further explore these structures.

III. RESULTS AND DISCUSSIONS

As mentioned earlier, it has been known for many years that the growth rate for compressible shear layers is much less than their incompressible counterparts. It is interesting to note how many different ways the thickness and growth rates of free shear layers have been defined. One of the thickness definitions is the vorticity thickness, which takes into account a majority of the points on the velocity profile as well as being a relatively simple calculation if the velocity profile is known. Bogdanoff used this definition to calculate the growth rate of the shear layer in the self-similar region and nondimensionalized it by its incompressible counterpart given, based on experimental results by Brown and

![Fig. 2. A schlieren photograph of the \( M_s = 0.86 \) flow field.](image)
Roshko. This nondimensional growth rate for all three convective Mach number cases of the present work showed very good agreement with those presented by Bogdanoff, which decrease with increasing convective Mach number.\textsuperscript{2,19,23}

Another use of the vorticity thickness is in defining a similarity parameter to collapse the velocity profiles in the self-similar region of the flow. A vorticity thickness similarity parameter defined as \( y^* = (y - y_{0.5}) / \delta_u \), where \( y_{0.5} \) is the location of \( U = (U_1 + U_2)/2 \) and \( \delta_u \) is the vorticity thickness. It was found that the vorticity thickness based similarity parameter not only collapses the velocity profiles in different streamwise locations, but also accounts for the effects of compressibility and collapses mean velocity profiles for different convective Mach numbers on a curve such as \( u^* = 0.5(1 + \tanh(2.4y^*)) \).\textsuperscript{2,19,23} Also, it was found that the momentum thickness-based similarity parameter needs a convective Mach-number-dependent coefficient to collapse the velocity profile for different convective Mach numbers.\textsuperscript{2}

One question that arises in evaluating fully developed compressible free shear layers concerns the onset of self-similarity of the shear layer. The self-similar region (or fully developed region) has been investigated extensively for incompressible shear layers, but lacking for compressible shear layers. Some investigators have reported the beginning of the fully developed region by a number of incoming boundary layer momentum thicknesses (\( \chi / \Theta \)). The location of self-similarity has been proposed to be anywhere from 350 to 1000 momentum thicknesses from the splitter plate trailing edge.\textsuperscript{8,10} Chinzei et al.\textsuperscript{13} reported that the self-similar location moves farther downstream as the velocity gradient across the shear layer decreases. The uncertainty of the initiation of the self-similar region has been suggested as a cause for experimental scattering in the spread rate parameter and the growth rate calculations.\textsuperscript{3} Metha and Westphal\textsuperscript{7} proposed three criteria for the onset of the fully developed region: (1) The mixing layer thickness grows linearly with the streamwise distance; (2) the shape of the mean velocity profile is independent of downstream distance when scaled by the local mixing layer thickness; and (3) the profiles of all turbulence quantities are independent of streamwise location when scaled by mixing layer thickness; in particular, peak values of the turbulence stresses should be independent of the streamwise location. Although other investigators have proposed and discussed similar conditions for the self-similarity region, many experimentalists have used the first two conditions because of the lack of turbulence data. Self-similarity of the turbulence profiles were checked for all convective Mach numbers presented here, but we will discuss only the trends for case 1 as a sample.

The development of self-similarity of the mean velocity profiles have been previously shown (Fig. 5 of Ref. 2). The velocity profiles seemed to collapse relatively well in the subsonic side of the shear layer, but the supersonic side became self-similar more gradually. It was tempting to include the profile at \( x = 120 \) mm in the fully developed region, but as will be seen by the turbulence results this location was within the developing region.

In Figs. 3–5 we present the development of the streamwise and lateral turbulence intensities and the Reynolds stress, respectively. Turbulence intensity is defined as the standard deviation of either the streamwise or the lateral velocity fluctuations normalized by the velocity difference of the two free streams. The maximum streamwise turbulence intensity decreases approximately from 19% to 16.5% as the flow becomes fully developed. Also, it is interesting to note that the lateral extent of the turbulence seems to decrease as the shear layer proceeds to a fully developed state, especially for the subsonic stream side. The lateral turbulence intensity profiles (Fig. 4) show the same decreasing trends in the lateral extent as the shear layer approaches self-similarity, but the maximum levels are approximately constant as the flow becomes fully developed. Incompressible shear layers, such as the unforced cases of Oster and Wignaski\textsuperscript{16} have similar trends in turbulence level and lateral extent as the flow becomes fully developed. The Reynolds stress profiles show a

![FIG. 3. Streamwise turbulence intensity in the developing and fully developed regions.](image-url)
decrease in level and lateral extent of the turbulence (mostly large-scale fluctuations) within the shear layer as it becomes fully developed. This is similar to the subsonic case of Hussain and Hussain who attribute this to the breakdown of large-scale structures following roll-up and pairing processes. Note that all turbulence values presented here asymptotically tend to free-stream values on both sides of the shear layer.

The collapse of mean velocity profiles with different convective Mach numbers indicated that \((U_i - U_j)\) and \(y^*\) are proper scaling parameters and these parameters were used to compare the turbulence characteristics of compressible and incompressible shear layers and to determine the effects of convective Mach number on the turbulence characteristics.\(^{2,19,23}\) Mean flow and turbulence results showed that cases 1 and 2 were fully developed for \(x > 150\) mm, and case 3 for \(x > 180\) mm. Since the turbulence profiles for each streamwise location collapsed (relatively well) in the self-similar region, only the averaged results will be shown for each convective Mach number. This averaging simplifies the graphs presented here and perhaps also reduces the uncertainty resulting from the smaller sample size. Reference 19 gives the individual profiles in the self-similar region of all three cases presented here.

Figure 6 shows the streamwise turbulence fluctuations profiles for the present experiments and the incompressible results of Oster and Wygnanski\(^{10}\) in the self-similar region. The streamwise turbulence shows that the compressibility reduces the maximum fluctuations that occur around \(y^* = 0\) for both compressible and incompressible flows. The maximum fluctuations for incompressible flows, shown in Fig. 6, are about 18.5%, very close to values reported by other investigators.\(^8,9\) The maximum drops to 16.5% for \(M_c = 0.51\), 15% for \(M_c = 0.64\), and 13% for \(M_c = 0.86\). In any given \(y^*\) location, especially in the high-speed side of the flow, the intensity level increases as the convective Mach number decreases and it seems that the incompressible results define the upper limit of the fluctuations. Another way of viewing this is to say that the lateral extent of the streamwise turbulence decreases with increasing convective Mach number.
(i.e., less of the thickness of the shear layer is turbulent as the convective Mach number increases). Figure 7 shows the lateral turbulence fluctuation with trends similar to those of the streamwise turbulence fluctuations. The lateral turbulence fluctuation of the subsonic case is much higher throughout the shear layer than the supersonic cases presented here. The subsonic results from Mehta and Westphal\textsuperscript{17} show levels and trends similar to those of Oster and Wygnanski.\textsuperscript{10} This indicates that even though the effect of compressibility on the mixing layer growth rate starts around $M_c = 0.5$, the effect on the turbulence fluctuations (especially the lateral) starts at much lower convective Mach numbers. It is desirable to compare these results to those obtained at higher convective Mach numbers, however, the available results that are measured directly at higher convective Mach numbers are mostly obtained with the presence of recirculation flows.\textsuperscript{25,26} These recirculating flows are returning from highly energetic reattaching regions and interacting with the shear layer and producing higher intensities.

Figure 8 shows the Reynolds stress distributions for the present experiments and the subsonic results of Oster and Wygnanski for a velocity ratio of 0.6. The difference between the subsonic and supersonic results are similar to those of the lateral turbulence intensity case. The maximum Reynolds stress values for case 1 are about 40\% lower than the subsonic results, the other supersonic results decreasing further. This is another indication that the effects of compressibility on the turbulence field starts at much lower convective Mach numbers. Figures 7 and 8 show a general trend that the level and lateral extent of both small- and large-scale fluctuations decrease with increasing convective Mach number in the high-speed side of the flow. The maximum values of the Reynolds stress and the turbulence intensities are plotted in Fig. 9. It is interesting to note that the maximum levels of the turbulence fluctuations and Reynolds stress decrease almost linearly with increasing $M_c$.

It may seem surprising that the turbulence properties in the self-similar region do not collapse, especially the Rey-
nolds stress, since it can be calculated (with some approximations) from the velocity profiles that do collapse relatively well. Lumley\textsuperscript{27} investigated a similar paradox in thermal mixing layers with the relationship between the temperature profile and the heat flux (mathematically similar to the velocity profile and Reynolds stress). He found that a very small fourth-order component of temperature variance has a surprisingly large effect on the maximum value of the heat flux. In a supersonic boundary layer with large distributed surface injection, Fernandez and Zukoski\textsuperscript{28} calculated the shear stress distribution from the velocity profile. The analysis is similar for the present two-stream planar shear layer. With a frame of reference moving with the bottom stream at a velocity $U_1$, the two-dimensional continuity and momentum equations in a zero pressure gradient self-similar region are given as

$$\frac{\partial(\rho U^*)}{\partial x} + \frac{\partial(\rho V^*)}{\partial y} = 0,$$  \hspace{1cm} (3)

$$\frac{\partial(\rho U^*)}{\partial x} + \frac{\partial(\rho V^*)}{\partial y} = 0.$$

Integrating both equations and combining them with the assumptions used by Fernandez and Zukoski\textsuperscript{28} (that in the self-similar region $\rho(U - U_2)/\rho_1(U_1 - U_2) = f(y/\Theta)$ and that the entrainment rate is approximately equal to the momentum thickness growth rate $(d\Theta/dx)$), an equation is obtained for the shear stress,

$$\frac{\tau(y)}{\rho_1(U_1 - U_2)} = \frac{\partial\Theta}{\partial x} \left\{ \frac{U^*}{U^*} \int_0^{y/\Theta} \frac{\rho}{\rho_1} U^* d\left( \frac{\bar{P}}{\Theta} \right) \right\} + \int_0^{y/\Theta} \frac{\rho}{\rho_1} U^* d\left( \frac{\bar{y}}{\Theta} \right),$$

where $\bar{y}$ is any given point in the shear layer to be calculated.

Figure 10 shows the shear stress calculated from the velocity profiles for the three convective Mach number cases. The trend that the level and extent of the shear stress decreasing with increasing convective Mach number is con-

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig8}
\caption{Reynolds stress profiles in the fully developed region.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig9}
\caption{Maximum values of turbulence intensities and Reynolds stress.}
\end{figure}
consistent with the direct Reynolds stress measurements (Fig. 8). Investigating Eq. (5), the shear stress basically equals the momentum thickness growth rate multiplied by a velocity-dependent integral quantity that collapses relatively well with different convective Mach numbers. Thus the observed decreasing trends are due mainly to the decrease in $d\Theta/dx$ as the convective Mach number is increased. In an earlier paper,\(^2\) it was found that the local momentum thickness in the shear layer can be used to collapse the velocity profiles only if it was multiplied by a convective Mach-number-dependent parameter, which is believed to cause the trend seen in the calculated shear stress.

In order to have a little more direct comparison of the measured Reynolds stress and the calculated Reynolds stress shown in Fig. 10, the results in Fig. 8 were multiplied by the density ratio $\rho/\rho_1$. The average density was determined from the measured mean velocity, static pressure, and stagnation temperature. Figure 11 shows the results. Comparing the results in Figs. 10 and 11, not only do they show similar trends, but the levels also agree well. We need to note that in compressible flow the Reynolds stress has two other terms besides the one shown in Fig. 11.\(^2\) The other two terms represent density–velocity fluctuation correlations. Following Morkovin’s hypothesis that the effects of density fluctuations on turbulence are negligible if the root-mean-square density fluctuations are small relative to the mean density, these two terms have generally been ignored.\(^4\) Bradshaw suggests a Mach number of 5 as the upper limit of Morkovin’s hypothesis for boundary layer and wake flows.\(^5\) A good comparison of the results in Figs. 10 and 11 seems to show that Morkovin’s hypothesis is valid in this type of flows as well.

One of the concerns of using LDV in high-speed flows is whether the LDV seed particles would follow a relatively high frequency of the large energy-containing eddies. A good comparison between results in Figs. 10 and 11 is a good indication that the silicone oil particle of approximately 1 $\mu$m diameter are following large-scale structures. In fact, recent direct numerical simulations of particles with inertia in compressible shear layers strongly support these findings.\(^3\)

Figures 12 and 13 present the streamwise and lateral
skewness, respectively. The sign of the skewness gives the side of the "tail" of the probability density function (PDF); a negative skewness means that the tail is on the low-speed side of the PDF relative to the mean, a positive skewness means that the tail is on the high-speed side of the PDF. Skewness has a value of zero for a normal distribution. The skewness shows the intermittency within the shear layer caused by the large-scale structures perturbing into the free streams. The \( u \) component of skewness has three zero locations: at each free stream and at the center of the shear layer. The profile has positive skewness on the subsonic side of the shear layer and a negative skewness on the supersonic side. The trend shown here is similar to those found in other compressible and incompressible shear layers,\(^9,25\) but the incompressible results show a much higher magnitude on the low-velocity side (2.5 times higher). The present results show that in the supersonic side of the shear layer, the magnitude of the skewness generally decreases with increasing convective Mach number (the lower two being relatively close). The magnitudes of the peaks of the \( u \) component of skewness are also about 30% higher in incompressible shear layers.\(^9,12,19\)

Flatness for the \( u \) component is given in Fig. 14. The flatness indicates whether there are high probabilities in the PDF profile far away from the mean. Thus it indicates the probability of large fluctuations, relative to the mean. A skewed tail, or existence of different high probability modes within the PDF profile, will result in a high flatness. The value of flatness for a normal distribution is 3, as defined normally. The \( u \) component of flatness shows peaks at the edges of the shear layer with a value of approximately 3 in the center and at both free streams. The incompressible shear layers have the same trend, but peak on the high-velocity side at about 8 and on the low-velocity side at about 22.\(^9\) Two things are observed from the \( u \) component of flatness in the present results. First, if the two peaks show the edges of the shear layer, the thickness decreases with increasing convective Mach number. Second, the level of the peak flatness on the high-speed (supersonic) side of the shear layer generally decreases with increasing convective Mach number. The
decrease could be the result of a suppression in the intermittency from large-scale structures believed to extend into the free-stream engulfing fluid at the edges of the shear layer. This is in agreement with the smaller growth and entrainment rates observed at higher convective Mach numbers. The narrowing of the large-scale structures within the shear layer and their lower magnitude shows how this engulfing entrainment is suppressed. The $v$ component of flatness shows similar characteristics as those described above for the $u$ component.

It should be remembered that the higher-order moments (skewness and flatness) results are based on a low sample size, however, the streamwise profiles averaging would perhaps reduce this uncertainty to some degree. The higher-order fluctuation results (even the Reynolds stress and lateral turbulence fluctuation results) for the two lower convective Mach number cases ($M_c = 0.51$ and $0.64$) seem to be very close together, however, a significant change in characteristics occur between these and the last convective Mach number ($M_c = 0.86$). The former seems to be due to the closeness of the two lower convective number cases, where the statistical uncertainty in the higher-order fluctuations could mask the close differences resulting from the compressibility effects.

IV. CONCLUSIONS

Experimental investigations of high Reynolds number compressible free shear layers are presented to explore the effects of compressibility on the turbulence field. Previous preliminary results showed that the level and lateral extent of turbulence fluctuations are reduced as the convective Mach number is increased. The present work extended the convective Mach number to $M_c = 0.86$, which showed the same trends, but the effects of compressibility are much more pronounced. The shear stress profiles calculated from the experimental mean velocity profile show a strong dependence on $M_c$ with trends and values similar to the experimental Reynolds stress. Also, higher-order moments of turbulence fluctuations such as skewness and flatness are reported. These quantities indicate the intermittency of the fluctuations resulting from large-scale structures at the edge of the shear layer. The level and lateral extent of the skewness and flatness were reduced with increasing convective Mach numbers. This is believed to be a reason for the lower entrainment and growth rates for compressible shear layers. Development of the mean flow and turbulence fluctuation profiles were found to have similar trends, as seen in incompressible shear layers.

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