The characteristics and evolution of large-scale structures in compressible mixing layers

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Single- and double-pulsed visualizations were employed to study the characteristics and evolution of large-scale structures in compressible mixing layers with convective Mach numbers \( (M_c) \) of 0.51 and 0.86. Instantaneous images and spatial correlations based on large ensembles of images show that large-scale structures which span the entire mixing layer thickness are a more dominant feature at \( M_c = 0.51 \) than at \( M_c = 0.86 \). Double-pulsed images in the \( M_c = 0.51 \) developing region show the structures’ formation to proceed as the roll-up of a wavy mixing region, in agreement with the Kelvin-Helmholtz picture of structure formation derived from studies of incompressible mixing layers. However, little indication of roll-up was present for \( M_c = 0.86 \). In the \( M_c = 0.51 \) fully developed region, the large-scale structures evolve similarly to those of incompressible mixing layers, even undergoing pairing processes. Similar structure evolution processes were not encountered in \( M_c = 0.86 \); instead, it was more difficult to even track large-scale motions between the initial and delayed images. Convective velocities derived from space-time correlations of ensembles of double-pulsed images are in good agreement with the theoretical convective velocity at the center of the mixing layer, but are higher than the theoretical value toward the high-speed side.

I. INTRODUCTION

The compressible planar free mixing layer has undergone much study in the past two decades. Two parallel streams of air, at least one of which is supersonic in the compressible case, combine downstream of a splitter plate to form a shear layer that grows in thickness with increasing downstream distance. It has been known for many years that compressible mixing layers have a lower growth rate than their incompressible counterparts. Brown and Roshko\(^1\) studied the incompressible shear layer between streams of different densities and showed that although the density ratio plays a role in reducing the growth rate, it is not significant enough to account for the large reductions in compressible mixing layers’ growth rate. The experimental results of Papamoschou and Roshko\(^2\) and theoretical results of Bogdanoff\(^3\) showed compressibility effects are primarily responsible. In an attempt to identify a compressibility parameter, the convective Mach number \( (M_c) \), defined as the Mach number in a frame of reference moving with the large-scale structures within the mixing layer, was developed by Papamoschou and Roshko.\(^2\) Since then, the convective Mach number has been shown an effective parameter for correlating decreases in growth rate, Reynolds stresses, and spatial correlation levels with increasing compressibility in both experimental\(^4-6\) and computational studies.\(^7-10\)

One topic that has not been addressed by measurements of turbulence quantities or growth rate and remains poorly understood is the characteristics of the turbulent structures within the mixing layer. Hussain\(^11\) defines a coherent structure in a turbulent flow as “a connected turbulent fluid mass with instantaneously phase-correlated vorticity over its spatial extent;” in other words, a region of large-scale vorticity distinct from the surrounding turbulence. Brown and Roshko\(^1\) were among the first to discover large-scale coherent structures in the incompressible planar mixing layer. The two-dimensional structures, which are aligned in the spanwise direction and possess distinct core and braid regions, have been called Brown- and Roshko-type or roller-type structures. With plan view schlieren photographs, Bernal and Roshko\(^12\) found well-defined streamwise streaks on the large-scale structures, suggesting the presence of streamwise vortices intertwined on the larger spanwise rollers. These streamwise vortex structures were also observed by Jimenez.\(^13\)

The initiation of the large-scale structures has been related to Kelvin–Helmholtz instability waves starting from the tip of the splitter plate.\(^12\) These instability waves are generally associated with upstream disturbances, which are amplified at the most unstable wavelengths, as derived from linear stability theory.\(^14\) Winant and Browand\(^15\) describe the initiation of roller structures from a constant vorticity layer between two parallel streams. The perturbations from the initial small amplitude wave induce transverse velocities in the layer, causing the vorticity containing region to become periodically fatter and thinner. The vortical regions are eventually pinched off to form the well-defined roller structures.

In general, the large-scale structures do not simply convect downstream, but instead undergo significant evolution processes as they convect. Three such structure evolution processes as presented by Hussain\(^11\) and suggested by other incompressible investigations\(^16-18\) are entrainment, pairing, and tearing. The rotation of the large-scale structures gives rise to the entrainment of free-stream fluid.\(^17,18\) The added fluid results in an enlargement of the entraining structure. Pairing occurs when two vortices approach each other and begin to rotate around a common origin as they combine into a single structure.\(^17\) This can occur with participation of the entire structure or a fraction of a structure.\(^11\) By appropriately forcing the incompressible mixing layer, it is possible...
to enhance or inhibit the pairing of adjacent structures. For example, three or more structures were seen to pair in the forced mixing layer of Ho and Huang. Tearing occurs when a structure is torn into two or more parts that become independent structures.

Although a wealth of information exists about the large-scale structures of the incompressible mixing layer, our knowledge of the structures in the compressible mixing layer is much less complete. Papamoschou and Roshko, using schlieren photography, were the first to demonstrate the presence of large-scale structures in the compressible planar mixing layer. Their presence has since been confirmed by others using laser-based visualizations.

Planar laser induced fluorescence from nitric oxide seeded into the low speed free stream was used by Clemens et al. to investigate the mixing layer structures. Statistical characteristics of the mixture fraction such as mean, RMS fluctuations and PDFs were computed. For $M_c = 0.28$, the RMS fluctuations were about 15% higher and the PDFs were typically broader than for $M_c = 0.62$, suggesting the $M_c = 0.28$ large-scale structures are more dominant than the $M_c = 0.62$ structures. Clemens and Mungal collected the light scattered from condensed ethanol to investigate changes in structure characteristics for a range of convective Mach numbers. Distinct core and braid regions similar to those of incompressible mixing layer structures were found for convective Mach numbers less than approximately 0.50. For higher convective Mach numbers, the structures became more three dimensional. Messersmith et al. also employed condensed water particles for visualizations of the shear layer. Two-dimensional covariance fields were calculated for the purpose of investigating structure size, shape, and eccentricity. An increase in size and eccentricity and a reduced angular orientation (relative to the streamwise direction) were encountered with increasing convective Mach numbers. Elliot et al. employed condensed water particles (formed when water vapor in the low speed stream mixed with the low temperature, dry air of the supersonic stream) to visualize the large-scale structures of $M_c = 0.51$ and 0.86 mixing layers. Similar to the results of Clemens and Mungal, the structures of the $M_c = 0.51$ case possessed core and braid regions similar to incompressible structures. However, the $M_c = 0.86$ mixing layer structures were more three-dimensional and generally less organized. The two-point correlation measurements of Samimy et al. showed that the $M_c = 0.86$ structures tend to be spanwise oblique; a finding consistent with stability analyses and simulations.

Although much in the way of instantaneous flow visualization has been done to document the existence and appearance of large-scale structures in the compressible mixing layer, little is known about the evolution of the structures. Double-pulsed schlieren images have been used to calculate convective velocities. Double-pulsed visualizations have also been used to investigate the convective velocity of large-scale structures in the axisymmetric mixing layer surrounding a supersonic jet and in the compressible planar mixing layer. However, these studies focused on investigating the convective Mach number concept and not the evolution of large-scale structures.

### Table I. Mean flow parameters.

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_c$</th>
<th>$M_e$</th>
<th>$U_1/U_e$</th>
<th>$\rho/\rho_1$</th>
<th>$\delta$ (mm)</th>
<th>$\theta$ (mm)</th>
<th>$Re_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51</td>
<td>1.8</td>
<td>0.36</td>
<td>0.64</td>
<td>8.0</td>
<td>0.62</td>
<td>27700</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0.25</td>
<td>0.25</td>
<td>0.37</td>
<td>9.2</td>
<td>0.51</td>
<td>24700</td>
</tr>
</tbody>
</table>

The purpose of the present study is to investigate the evolution and characteristics of the large-scale structures in $M_c = 0.51$ and 0.86 mixing layers. This is accomplished by processing statistically significant samples of single- and double-pulsed instantaneous visualizations.

### II. EXPERIMENTAL ARRANGEMENT

The experiments were conducted at the Aeronautical and Astronautical Research Laboratory of The Ohio State University. A 3.18 mm thick steel splitter plate with a machined angle on the subsonic side of approximately $1^\circ$ over a 125 mm length and a flat profile on the supersonic side separates the two flows. The trailing edge of the splitter plate has a thickness of 0.5 mm. The upper stream is always supersonic, with two interchangeable converging-diverging nozzles with nominal Mach numbers of 2 and 3, resulting in mixing layers with convective Mach numbers of 0.51 and 0.86. The bottom stream is subsonic, with a Mach number that is set to provide a constant pressure mixing layer. The streams merge downstream of the splitter plate to form a planar shear layer in the 152.4X152.4 mm test section. A summary of flow parameters is given in Table I. In this table, $\delta$ and $\theta$ are the high speed incoming boundary layer thickness and the boundary layer momentum thickness, and the $Re_e$ is based on the velocity difference between the two streams and the high speed incoming boundary layer momentum thickness and density. This wind tunnel facility has been used in other investigations of compressible mixing layers.

A light sheet was created with a combination of cylindrical and spherical lenses using a Quanta Ray GCR-4 frequency-doubled (532 nm) Nd:YAG laser. The Nd:YAG laser has been modified to provide two pulses for each lamp excitation by multiple Q switching. The pulse width of both the initial and delayed pulses is 9 ns, which effectively freezes the flow field. Two Princeton Instruments intensified CCD cameras are synchronized to the laser, and the images are stored on two 486 PCs. This system allows two successive images to be taken with an adjustable time delay from 20 to 200 $\mu$s. The time delay is measured with an oscilloscope. Figure 1 presents the camera and light sheet orientation used in the current study. Optical access is provided by windows on the side, top, and bottom walls, with possible side viewing areas 80 mm high and 450 mm long, and top and bottom viewing areas 30 mm wide and 300 mm long. This allows both streamwise ($z$-$y$ plane) and plan view ($x$-$z$ plane) images to be collected. Scattered light is col-
lected from condensed water particles formed with the product formation technique. The low speed stream consists of moist ambient air. When the water vapor in the low speed stream mixes with the low temperature air of the supersonic stream, condensation is formed in the mixing layer. Concerns about the size of the condensation particles and the time scale for their formation have been previously addressed, and the particles are believed to accurately represent the features of the mixing layer.

III. RESULTS AND DISCUSSION

A. Single-pulsed images and statistics

Although qualitative observations made in conjunction with instantaneous single-pulsed flow visualizations have been reported by many investigators, few attempts have been made to ascertain the average characteristics of large-scale structures by processing significant numbers of visualizations. Such calculations were performed for ensembles of 400 streamwise images. Figure 2 gives single instantaneous streamwise views in the fully developed regions of the $M_c=0.51$ and 0.86 mixing layers, and the corresponding ensemble average and ensemble RMS fluctuation images. The flow direction in these and all other images is right to left. The instantaneous image of Fig. 2(a) ($M_c=0.51$) demonstrates the roller-type structures that have been observed at convective Mach numbers less than approximately 0.5.$^{22,23}$

As shown in Fig. 2(b), the structures in the $M_c=0.86$ mixing layer are less organized than those in the $M_c=0.51$ layer. The higher growth rate of the $M_c=0.51$ layer is evident in the ensemble averages of Figs. 2(c) and 2(d), even though $U_2/U_1$ is much smaller for the $M_c=0.86$ mixing layer (Table I). Mixing layer growth is more significant on the subsonic side of the mixing layers, which is consistent with earlier LDV results.$^5$ Each column of the average and RMS images has been nondimensionalized by the maximum average in that column, $I_{max}(x)$, which occurs at a lateral location $y_0$. This was done to remove the effects of the Gaussian intensity distribution of the interrogating laser sheet.

Although $I_{rms}/I_{max}$ is much larger for the $M_c=0.86$ mixing layer (Fig. 2), the regions characterized by significant RMS fluctuations are significantly wider than the visual thicknesses, $b$, obtained from the average images. Further, the significant RMS region exhibits a bimodal shape in which peaks occur both above and below the center of the mixing layer. This is true for both convective Mach numbers. In order to display profiles of the RMS fluctuations, a nondimensional lateral scale, given by

$$y^* = \frac{(y-y_0)}{b},$$

is used. The visual thickness, $b$, is calculated as the distance between the bottom and top edges of the shear layer, as defined by 15% of $I_{max}$. By looking at $I_{rms}/I_{max}$ profiles at various downstream locations (Fig. 3), the bimodal shape of the RMS profiles becomes more obvious. Although the streamwise and lateral turbulence intensities did not possess this bimodal profile,$^4$ the trends of decreasing level and lateral extent with increasing convective Mach number found previously in the turbulence intensity profiles are also present in the $I_{rms}/I_{max}$ profiles. The peaks and valleys for both convective Mach numbers occur at the same $y^*$. For both convective Mach numbers, the $I_{rms}/I_{max}$ peak on the high speed side is smaller than peak on the low speed side. The ratio of the peak on the high speed to the peak on the low speed side is approximately 0.9 for $M_c=0.51$. This disparity is more pronounced for $M_c=0.86$, as the ratio of the peaks has fallen to 0.75. This is probably due to less frequent excursions of large-scale structures into the supersonic free-
stream at $M_c=0.86$, an idea that is strongly supported by the instantaneous images [see Figs. 2(a) and 2(b)]. Further support is found in the streamwise and lateral flatness measurements of Elliott and Samimy,\textsuperscript{5} where a sharp reduction in the maximum level near the high speed edge of the mixing layer with increasing $M_c$ was again encountered. It is also possible that the $I_{RMS}/I_{max}$ results may be influenced by the dynamics of the condensation formation process.\textsuperscript{21} The bimodal RMS shape has been seen in temperature\textsuperscript{31} and mixture fraction measurements\textsuperscript{32} in incompressible mixing layers and in product formation measurements in compressible mixing layers.\textsuperscript{21} Clemens et al.\textsuperscript{24} observed the same bimodal shape for mixture fraction RMS results in laser induced fluorescence studies of compressible shear layers. Reduction of the peak on the high speed side with increasing $M_c$ was observed, but peaks were not as distinct as those of Fig. 3. It has been suggested that the essentially uniform scalar concentration of the large-scale structures is mainly responsible for the bimodal RMS profile,\textsuperscript{21} and that the reduction of the peak on the high-speed side with increasing convective Mach number is due to a decreasing occurrence of these structures.\textsuperscript{24} One statistical quantity that can give an idea of the nature of the large-scale structures in a mixing layer is the two-dimensional spatial correlation about a stationary reference point. The equations defining this correlation, as given by Miles and Lempert,\textsuperscript{33} are

\begin{equation}
I_{RMS}^*(x,y) = \frac{1}{n} \sum_{j=1}^{n} I_j(x,y),
\end{equation}

\begin{equation}
I_{RMS}(x,y) = \left( \frac{1}{n} \sum_{j=1}^{n} [I_j^*(x,y)^2] \right)^{1/2}.
\end{equation}

\begin{equation}
R_s(x_{ref},dx,y_{ref},dy) = \frac{(1/n) \sum_{j=1}^{n} [I_j^*(x_{ref},y_{ref})I_j^*(x_{ref}+dx,y_{ref}+dy)]}{I_{RMS}(x_{ref},y_{ref})I_{RMS}(x_{ref}+dx,y_{ref}+dy)},
\end{equation}

where $x_{ref}$ and $y_{ref}$ are the coordinates of the origin in the image, $I_j(x,y)$ is the instantaneous intensity in the image, $n$ is the number of images, $I_{RMS}^*(x,y)$ is the instantaneous intensity fluctuation about the local mean in the image, $I_{RMS}(x,y)$ is the RMS intensity fluctuation in the image, and $R_s(x_{ref},dx,y_{ref},dy)$ is the two-dimensional spatial correlation. This correlation expression is nondimensionalized such that $R_s(x_{ref},0,y_{ref},0)=1.00$. The flow direction in the correlation fields of Figs. 4 and 5 is left to right.

Figures 4 and 5 show the two-dimensional spatial correlations in the streamwise plane for the $M_c=0.51$ and 0.86 mixing layers, respectively, for $x_{ref}=200$ mm at various lateral locations. As shown in previous LDV results,\textsuperscript{5} $x_{ref}=200$ mm is in the fully developed region. Four hundred images were used in the calculations, as correlations obtained with ensembles of up to 1600 realizations were not noticeably different than those based on 400 images.\textsuperscript{34} For both convective Mach numbers, the contours of constant correlation are nominally circular around reference points located in the lower portions ($y_{ref}=-0.51$) and center ($y_{ref}=0.0$) of the mixing layer. For the lower convective Mach number, as the reference point is moved toward the supersonic side, the contours of appreciable correlation extend farther down into the mixing layer. This is probably due to the fact that when a large-scale structure is present on the high speed side of the $M_c=0.51$ mixing layer, it is robust enough to be "detected" in the lower speed side. However, the structures in the $M_c=0.86$ mixing layer are not coherent enough to have such an effect. Another characteristic is that the contours become slightly angled and elongated in the streamwise direction as the reference point is moved to the supersonic side of the mixing layer. The increased angle and elongation of the contours on the supersonic side is much more exaggerated for the higher convective Mach number mixing layer [Fig. 5(c)]. This probably indicates the structure angle that is imposed on the large-scale structures for this case.

B. Double-pulsed images and statistics

Single instantaneous images and statistics calculated from ensembles of such images are good for making general observations about large-scale structures, but do not give insight into structure evolution. Such questions can be answered only if the time history of the structures can be studied. Although it would be more desirable to acquire a full time history of individual structures, this is not possible with available instrumentation. Double-pulsed images, however, can be acquired by using a multiple Q-switched laser (as discussed above), in conjunction with two cameras in the arrangement shown in Fig. 1.

The time delay between the initial and delayed images of each double-pulsed image set is presented in terms of a non-dimensional convection distance given by

\begin{equation}
x_c = U_c \tau /b,
\end{equation}

where $\tau$ is the time delay, $b$ is the mixing layer thickness as defined earlier, and $U_c$ is the theoretical convective velocity of the large-scale structures, given by

\begin{equation}
U_c = (a_1 U_2 + a_2 U_1)/(a_1 + a_2),
\end{equation}

where $U_1$ and $U_2$ are the tree-stream velocities and $a_1$ and $a_2$ are the speeds of sound of the faster and slower streams, respectively.\textsuperscript{2} This non-dimensional parameter has been used by other investigators in studying space–time correlations.\textsuperscript{34}
FIG. 4. Streamwise two-dimensional spatial correlations for $M_* = 0.51$ at (a) $y^* = -0.51$, (b) $y^* = 0.0$, and (c) $y^* = 0.49$ ($x_{ref} = 200$ mm).

FIG. 5. Streamwise two-dimensional spatial correlations for $M_* = 0.86$ at (a) $y^* = -0.51$, (b) $y^* = 0.0$, and (c) $y^* = 0.49$ ($x_{ref} = 200$ mm).
TABLE II. Double-pulse time delays.

<table>
<thead>
<tr>
<th>(M_c) (dev.)</th>
<th>(x_c)</th>
<th>(\tau (\mu s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.51</td>
<td>1.4</td>
<td>38.1</td>
</tr>
<tr>
<td>0.51</td>
<td>0.64</td>
<td>34.7</td>
</tr>
<tr>
<td>0.51</td>
<td>1.18</td>
<td>63.7</td>
</tr>
<tr>
<td>0.51</td>
<td>1.71</td>
<td>92.6</td>
</tr>
<tr>
<td>0.86 (dev.)</td>
<td>1.5</td>
<td>30</td>
</tr>
<tr>
<td>0.86</td>
<td>0.64</td>
<td>25</td>
</tr>
<tr>
<td>0.86</td>
<td>1.18</td>
<td>45</td>
</tr>
<tr>
<td>0.86</td>
<td>1.71</td>
<td>65</td>
</tr>
</tbody>
</table>

The theoretical \(U_c\) and measured \(b\) (in the fully developed region, \(x_c=190 \text{ mm}\)) are 338 m/s and 18.2 mm for \(M_c=0.51\) and 428 m/s and 16.3 mm for \(M_c=0.86\), respectively. Table II gives the actual time delays and nondimensional convection distances for the cases presented here. For both convective Mach numbers, one set of data was acquired in the developing region (indicated by “dev.” in Table II) and three sets of data with different time delays were acquired in the fully developed region. Although only very few double-pulsed image pairs can be presented to illustrate the various evolution processes experienced by the mixing layer structures, it should be kept in mind that the presented discussion is based on the review of large ensembles (400 image pairs for each data set), and care has been taken to identify only those processes whose occurrence in the ensembles of image pairs was common.

1. Streamwise view double-pulsed images

Figure 6(a) shows double-pulsed streamwise images in the \(M_c=0.51\) developing region with \(x_c=1.4\). Again these images have been normalized by \(l_{max}(x)\) to remove the effects of the Gaussian laser sheet distribution. The roller-type structures identified in the single-pulsed images are seen to convect downstream between the initial image and the delayed image. Also, many image pairs suggest the structures form from the “roll-up” of an initial wave. For example, the structures labeled 6a' and 6a'' in Fig. 6(a) seem to be captured at different points in the roll-up process. Although the entire formation of a structure is not captured in the image pairs, the structures’ formation is similar to the roll-up of a Kelvin–Helmholtz wave, which occurs in incompressible mixing layers.

Figure 6(b) gives a double-pulsed image pair acquired in the \(M_c=0.86\) developing region with \(x_c=1.5\). Compared to the \(M_c=0.51\) developing region, large-scale motions were much less organized. This was indicated by the increased difficulty encountered in following large-scale structures from the initial to delayed image as the \(M_c=0.86\) structures underwent much more significant changes than the \(M_c=0.51\) structures for similar convective distances. In addition, little indication of a well-defined roll-up process was encountered. The region labeled 6b' in Fig. 6(b), which shows a wave simply convecting downstream with no noticeable roll-up, is typical of the image pairs obtained in the \(M_c=0.86\) developing region.

Image pairs obtained in the \(M_c=0.51\) fully developed region with three different time delays are presented in Fig. 7. For all three delays there are recognizable large-scale structures that grow and evolve as they convect downstream. Roller-type structures with distinct core and braid regions similar to those observed in other studies are present. The presence of smaller scale turbulent motions within the large-scale structures is clearly indicated. Four basic categories of processes have been used to describe the evolution of these large-scale structures: pairing (complete, partial, and fractional), tearing, normal growth, and cascading. The structures labeled 7b' and 7b'' in Fig. 7(b) appear to be pairing into a single larger structure. From observations of many double-pulsed image pairs, it appears that if a major portion of a structure is located more toward the high speed side of the mixing layer than the preceding structure, the lagging structure tends to close on the preceding structure and the two structures have a chance to pair. This “pre-pairing” orientation is illustrated in Fig. 7(f), where the structures labeled 7f and 7f' (7f' outside of the first frame) have moved closer together and the trailing structure is laterally higher (nearer the high speed free stream).

At other instants of time the roller-type structures seem to convect with little change in physical appearance. For example, the structure labeled 7e' in Fig. 7(e) seems to have only rotated as it traveled from the initial to the delayed image. Even large-scale structures that do not have distinct
core and braid regions remain correlated as they convect downstream, as in Fig. 7(d). Although instances of structure tearing were less common than structure pairing for the double-pulsed images of the $M_c=0.51$ mixing layer, some indications of tearing were encountered. For example, a structure appears to experience at least fractional tearing in the region labeled 7c'. In several instances, the stretching of the braid region between two adjacent roller-type structures was captured in the image pairs. Such an occurrence is illustrated in Fig. 7(a), where the braid region labeled 7a' appears to be stretched by the vorticity of the two adjacent rollers. If this region contains streamwise vorticity, the magnitude of the streamwise vorticity would be expected to increase due to vortex stretching, as reported in incompressible studies.\(^1\)

Brown and Roshko\(^1\) suggest a lifetime of the roller-type structures on the order of $4.3\delta_w$. For the $M_c=0.51$ mixing layer this would correspond to a nondimensional convection distance of $x_c\sim4.4$, which is larger than measured here. At the largest employed delay, $x_c=1.71$, the large-scale structures are still well defined and easily tracked from the initial to the delayed image in the vast majority of the acquired image pairs.

Double-pulsed side view images in the fully developed region are given in Fig. 8 for the $M_c=0.86$ mixing layer for the same dimensionless time delays as for the $M_c=0.51$ case (Fig. 7). For $x_c=0.64$, structures could be followed from the initial to the delayed image as in Figs. 8(a) and 8(b). For $x_c=1.18$ [Figs. 8(c) and 8(d)], some features of the mixing layer could be followed, particularly on the low speed side of the mixing layer. However, large-scale structures underwent significant changes between the initial and delayed images. As illustrated in Figs. 8(e) and 8(f), very few mixing layer features could be followed from initial to delayed image for the largest dimensionless time delay of $x_c=1.71$. The incompressible structure lifetime of Brown and Roshko\(^1\) translates into a dimensionless convection distance of $x_c\sim6.1$ in the present nomenclature, which is much greater than the $x_c=1.71$ by which most large-scale structures become essentially unidentifiable.

As reported earlier, there is little indication of well-defined roller-type large-scale structures, and the structures are relatively unorganized for the $M_c=0.86$ mixing layer. In contrast to the $M_c=0.51$ mixing layer, where structures could be tracked for even the largest time delays, it was common for the structures of the $M_c=0.86$ mixing layer to appear in only one image of the double-pulsed pair, which suggests a sharp reduction in structure lifetime. Such an occurrence is illustrated by the structure labeled 8e' in the delayed image of Fig. 8(c), which does not appear in the initial image. Calculations for individual double-pulsed images where roller-type structures were encountered suggest that, when they occur, the roller-type structures convect at near the theoretical convective velocity. One common occurrence for the $M_c=0.86$ case is a tearing or stretching process, exemplified by structures 8a' and 8b' in Figs. 8(a) and 8(b). In these image pairs (as well as many others not shown) the top portion of a large-scale structure is accelerated relative to the portion of the structure on the low speed side of the
mixing layer, thus stretching or tearing it apart. Another characteristic of the $M_c = 0.86$ case is that structures seem to “pull” low speed free stream fluid (identified as a region of no signal in the bottom of the mixing layer) into the mixing layer. The entrained fluid is then further pulled into the mixing layer as it experiences the local acceleration [events 8c’, and 8d’ of Figs. 8(c) and 8(d)].

In viewing image pairs of the $M_c = 0.86$ mixing layer like those of Fig. 8, it is much easier to follow distinguishing characteristics near the top or bottom of the mixing layer [such as the bottom of event 8d’ of Fig. 8(d)] than entire structures. Some investigators have calculated convective velocities from these localized characteristics. This can lead to erroneous measurements of the structure convective velocity, since these features move at near the mean velocity at that location.

2. Streamwise view space–time correlations

In order to study the evolution of the mixing layer, and obtain convective velocity information about the structures as a whole, space–time correlations between ensembles of initial and delayed images have been calculated for both convective Mach numbers at various time delays. Traditionally, space–time correlations are computed with multiple probes, where data is collected for long time sequences at a few spatial points. The correlations presented here are different in that data is collected at only two instants in time, but in a two-dimensional image. Each of the presented correlations is based on 400 pairs of initial and delayed images. Before the correlations were calculated, average and RMS fluctuation images were generated for both the 400 initial images and the 400 delayed images. The appropriate average image was then subtracted from each of the initial and delayed images, so that the correlations are of the fluctuations from the local mean. The correlations were nondimensionalized by the RMS fluctuation at the two pixel locations being correlated (the RMS at the reference location was taken from the initial RMS image and the RMS at the shifted location was taken from the delayed RMS image). In the initial fluctuation image, a vertical line is taken at a streamwise location $x_0$. The signal in this vertical line is then correlated with the corresponding signal in the delayed image at streamwise locations denoted by $x_0 + dx$. The resulting correlation is then averaged over the 400 image pairs in the ensemble for the given time delay. To reduce the dependence on the selection of the reference line location ($x_0$), the correlations for four different $x_0$ locations spaced through the initial images were averaged together. Calculations were made using more images and reference locations but the level of the correlation contours remains approximately the same. It also should be noted that the correlation program was run on ensembles of unrelated images and the resulting maximum correlation coefficient was less than 0.1.

The expressions for the space–time correlation are given by

$$I_{\text{RMS},ijkl}(x,y) = \frac{1}{n} \sum_{j=1}^{n} \left( I_{\text{ijkl}}(x,y) \right)^2$$

$$R_{s,-\mathcal{L}(x_{\text{ref}},dx,y)} = \frac{1}{I_{\text{RMS},ijkl}(x_{\text{ref}},dx,y)}$$

where $\tau$ is the time delay between the initial and delayed images, $I_{\text{ijkl}}(x,y)$ is the instantaneous intensity in the initial or delayed image, $n$ is the number of image pairs for the given time delay, $I_{\text{ijkl}}(x,y)$ is the instantaneous intensity fluctuation from the local mean in the initial or delayed image, $I_{\text{RMS},ijkl}(x,y)$ is the RMS intensity fluctuation in the initial or delayed RMS image, and $R_{s,-\mathcal{L}(x_{\text{ref}},dx,y,\tau)}$ is the space–time correlation calculated from the initial and delayed image. It should be noted that this correlation expression is nondimensionalized, such that $R_{s,-\mathcal{L}(x_{\text{ref}},dx,y)} = 1.00$. One should note that this correlation tracks the convection of a vertical line placed in the initial image, and in no way a reflection of the shape or appearance of the large-scale structures. Further, it should be kept in mind that since the lines are located at a fixed spatial location in the initial images (and not always located at the center of a structure), the correlation contours are not entirely a reflection of the convection of large-scale structures. However, it is expected that large-scale structures are mainly responsible for the correlation between initial and delayed images. The streamwise direction in the correlation contour plots is from left to right.

Figures 9 and 10 give the correlation contours for three nondimensional convective distances $x_c$ (i.e., time delays), for $M_c = 0.51$ and $M_c = 0.86$, respectively. For all of the contours, the point of highest correlation occurs between the spatial shifts expected from the lower and higher speed free-stream velocities and the employed time delay. A striking characteristic of the correlation contours presented here (Figs. 9 and 10) is that two maxima are found to occur, one near each edge of the mixing layer. The probable explanation lies in the nature of the turbulent mixing layer. Near the edges of the mixing region, large fluctuations occur about the mean (indicated by the RMS images of Fig. 2), which are associated with the passage of large-scale structures. The fluctuations associated with the structures appear to remain well correlated for the employed time delays. The higher correlations at the edges of the mixing layer relative to the center are thought to be a result of a wider range of scales contributing to the fluctuations at the center (i.e., smaller scales on average in the center relative to the edges) and correlation levels tend to increase with the scale of the motions.

Convective velocities can be derived from the correlation contours via the known time delay between the double-pulsed images. These velocities were calculated for the points of maximum correlation near the top and bottom of the mixing layer and the maximum correlation point at $y = 0.0$. The velocity derived from the point in the middle of the mixing layer ($y = 0.0$) is 309 m/s, which is slightly less than the theoretical convective velocity. However, the deviation between the two is within the accuracy of the mean-
FIG. 9. Streamwise two-dimensional space–time correlation fields in the fully developed region of the $M_c=0.51$ mixing layer for dimensionless convection distances ($x_c$) of (a) 0.64, (b) 1.18, and (c) 1.71.

Measurement and repeatability of the free-stream velocity due to slight temperature changes from day to day. The velocities derived from the local maxima near the bottom and top edges of the mixing layer (226 and 396 m/s) are lower and higher than the convective velocity, respectively. It should not be surprising that there is a lateral variation in convective velocity, since such a variation is required for the pairing and tearing observed in the double-pulsed images to occur. The lateral variation was also observed in two-point fluctuating pressure measurements, where velocities were derived from space-time correlations.\textsuperscript{25}

If one imagines an axis between the two regions of...
maximum correlation, it appears that the axis rotates and stretches from one time delay to another. The axis rotation gives an indication of the vorticity associated with the large-scale structures. It is difficult to identify the specific cause of the axis stretching. Contributing factors could be (1) a structure "tearing" process and (2) the reference lines for the correlations are not chosen a priori to fall within a structure so that portions of the reference lines which do not fall within a structure travel near the local free-stream velocity.

Figure 10 shows the correlation contours for $M_c = 0.86$. Figures 10(a)–10(c) are for the same $x_c$ values as the $M_c = 0.51$ contours of Figs. 9(a)–9(c), respectively. Again the velocity derived at the point in the middle of the mixing layer (373 m/s) is lower than the theoretical convective velocity (428 m/s), while the velocities derived from the two correlation maxima are lower and higher (272 m/s for the bottom maximum and 481 m/s for the top maximum). The correlation level has dropped for $M_c = 0.86$ relative to $M_c = 0.51$, suggesting the large-scale structures are less coherent. This is supported by double-pulse images with $x_c = 1.71$, where it is much more difficult to follow structures between the initial and delayed images for $M_c = 0.86$ than $M_c = 0.51$, particularly events on the high speed side of the shear layer. Assuming that the highest correlation levels are associated with the large-scale coherent motions, the "average" structure appears to be stretched, particularly on the high speed edge of the mixing layer where the structure appears to be tilted and elongated in the streamwise direction.

Figure 11 shows the maximum correlation coefficient (for the local maximum near each side of the mixing layer) plotted against dimensionless convection distance for both convective Mach numbers. It can be seen that the levels decrease much more quickly for $M_c = 0.86$ than $M_c = 0.51$, indicating the $M_c = 0.86$ structures have a shorter lifetime. This agrees with the results of multipoint correlations of fluctuating pressures, where it was found that the streamwise correlation levels decreased much more rapidly with increasing probe separation for $M_c = 0.86$ than for $M_c = 0.51$.25 The increase in the distance between the two local correla-

FIG. 11. Maximum correlation coefficient versus dimensionless convection distance for both sides of the $M_c = 0.51$ and 0.86 fully developed mixing layers.

3. Plan view double-pulsed images

Plan view double-pulsed images for the $M_c = 0.51$ and $M_c = 0.86$ mixing layers are shown in Figs. 12(a) and 12(b), respectively. Both images of Fig. 12 were taken with the laser sheet passing through the bottom edge of the mixing layer with a nondimensional convection distance of 1.18. As in the single-pulsed results of other investigators,22,23 two-
dimensional features are found in many of the images taken of the \( M_c = 0.51 \) mixing layer (for example, the condensation boundary that convects past the center of Fig. 12(a) between the initial and delayed image is aligned in the spanwise direction). These two-dimensional features are the result of the large-scale rollers, but the structure “fronts” typically display the presence of smaller scales as well. The structures remain essentially two dimensional in the initial and delayed images and convect slightly faster or slower than the theoretical \( U_c \) depending on the lateral location in the mixing layer (faster than \( U_c \) on the high speed side, and vice versa). Plan views through the high speed side of the \( M_c = 0.51 \) mixing layer also show the convection of two-dimensional structures. The \( M_c = 0.86 \) mixing layer structures are typically not as two dimensional as the \( M_c = 0.51 \) structures, but instead are highly three dimensional and often have oblique orientations of \( \pm 45^\circ \). Such an oblique structure is illustrated in Fig. 12(b). Again, the oblique structures convect slightly faster or slower than the theoretical \( U_c \), depending on the lateral location in the mixing layer. For the employed time delays, the oblique structures maintain a constant angle as they convect downstream. In the middle of the \( M_c = 0.51 \) and 0.86 mixing layers \( (y^* = 0) \), smaller scales partially mask the large-scale structures, making it more difficult to track the structures from the initial to the delayed image.

4. Plan view space–time correlations

Similar to the side view cases, space–time correlation contours were calculated for the plan views, and are given in Figs. 13 and 14 for the \( M_c = 0.51 \) and 0.86 mixing layers, respectively. Correlation contours are based on 300 images. In general, the velocities derived from the centroid of the contours is close to the local mean velocity at the top and bottom edges of the mixing layer, and close to the theoretical convective velocity in the middle \( (y^* = 0) \) of the mixing layer. The velocities calculated from the streamwise displacement of the point of maximum correlation for the \( M_c = 0.51 \) mixing layer are 243, 360, and 416 m/s for lateral planar cuts at \( y^* = -0.3, 0.0, \) and 0.4. For the \( M_c = 0.86 \) mixing layer the calculated velocities are 256, 404, and 507 m/s for planar cuts at \( y^* = -0.4, 0.0, \) and 0.4. This lateral variation of the convective velocity is consistent with the results of the streamwise views.

One difference between the two convective Mach numbers is the structure length in the streamwise direction \( (L_{pe}) \), which is defined as the streamwise extent of the region possessing correlation levels of at least 0.55 at the spanwise center of the tunnel. For the \( M_c = 0.51 \) mixing layer, \( L_{pe}/b \approx 0.68 \), and for \( M_c = 0.86, L_{pe}/b \approx 0.34 \). This suggests the two-dimensional roller-type structures of the \( M_c = 0.51 \) mixing layer remain more coherent and have a longer streamwise extent than the less organized structures for the \( M_c = 0.86 \) mixing layer. Although one might anticipate oblique correlation contours for the \( M_c = 0.86 \) mixing layer, it must be kept in mind that the correlation contours are a reflection only of the convection of a spanwise line, and not the shape or orientation of the large-scale structures. Since even the oblique structures were seen to convect at a constant angle, the elliptical contours of Fig. 14 are not surprising.

IV. CONCLUSIONS

Single- and double-pulsed visualizations were employed to study the characteristics and evolution of large-scale structures in compressible mixing layers with convective Mach numbers \( (M_c) \) of 0.51 and 0.86. Spatial correlations of large ensembles of single-pulsed visualizations were performed with the reference point centered at several lateral locations in the mixing layer. A more significant spatial extent of the region possessing significant correlation levels for \( M_c = 0.51 \) than for \( M_c = 0.86 \) shows that large-scale structures which span the entire mixing layer thickness are more dominant feature at \( M_c = 0.51 \). Although complete visual histories of individual structures were not available, double-
In the $M_c = 0.5$ developing region, double-pulsed results showed the large-scale structures to evolve very much like the structures of incompressible mixing layers. Velocities derived from the result- ing two-dimensional contours are approximately equal to the theoretical convective velocity in the center of the mixing layer. The derived velocities are higher and lower than the theoretical value on the high and low speed sides of the mixing layer, respectively.

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G. S. Elliott, M. Samimy, and S. A. Arnette, "Study of compressible


