Excitation of Free Shear-Layer Instabilities for High-Speed Flow Control

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Free shear layers are building blocks of many flows of interest in applications, including jets, cavity flows, and separated flows. It was found several decades ago that free shear layers are unstable to small perturbations over a wide range of frequencies, that they are dominated by coherent large-scale structures (even at high Reynolds numbers), and that the dynamics of these structures dominate important processes: entrainment, mixing, momentum transport, and noise generation. These findings motivated extensive research activities to actively control their development using excitation of instabilities, but early experimental research focused primarily on the control of low-speed, low-Reynolds-number free shear layers. This extensive body of the literature is briefly reviewed. The authors’ recent work using localized arc filament plasma actuators in jets shows that free shear layers respond to the excitation over a large range of conditions that have been explored: jet Mach number (up to 1.65), convective Mach number (up to 1), and Reynolds number (up to \(1.65 \times 10^6\)). However, the nature of large-scale structures, shear-layer growth rate, and generation of Mach waves all depend on the jet Mach number and compressibility level. The results clearly demonstrate the similarity of instability processes and development of large-scale structures in free shear layers, regardless of the Mach or Reynolds numbers.

Nomenclature

\[ D = \text{nozzle exit diameter} \]
\[ f = \text{frequency} \]
\[ m = \text{azimuthal mode number} \]
\[ M = \text{Mach number} \]
\[ R = (U_1 - U_2)/(U_1 + U_2), \text{normalized freestream velocity difference} \]
\[ r = \text{radial coordinate} \]
\[ St = \text{Strouhal number} \]
\[ t = \text{time} \]
\[ U = \text{streamwise velocity} \]
\[ x = \text{streamwise distance} \]
\[ y = \text{vertical distance} \]
\[ \alpha = \text{wave number [used in Eq. (1)]} \]
\[ \delta = \text{jet width at half centerline velocity} \]
\[ \theta = \text{local momentum thickness} \]
\[ \lambda = \text{structure wavelength} \]
\[ \omega = 2\pi f, \text{angular velocity} \]

Subscripts

\[
\text{ave} = \text{average} \quad [U_{\text{ave}} \text{ is equal to } (U_1 + U_2)/2] \\
D = \text{nozzle exit diameter} \\
e = \text{excitation (frequency or Strouhal number)} \\
j = \text{jet (diameter, frequency, or speed of sound)} \\
n = \text{most-amplified (frequency or wavelength)} \\
p = \text{preferred mode (frequency or Strouhal number)} \\
r = \text{response (frequency)}
\]

I. Introduction

FREE shear layers (FSL) are present in many flows. This class of flows, which develops away from surfaces that would impose a no-slip boundary condition, is ubiquitous in practical applications. They include, among others, jets, cavity flows, and separated flows. The presence of large-scale structures (LSS) in turbulent flows in general, and FSL in particular, has long been known. An example is Leonardo da Vinci’s sketch of a free water jet issuing from a channel into a pool and his observation and description of large- and small-scale structures in the 1500s. A similar, but poetic, description of large- and small-scale structures, as well as energy transfer from the former to the latter, was given by Richardson in the 1920s [1]. Later on, Corrsin [2] and Townsend [3] attributed the corrugating interface between the turbulent and nonturbulent parts of FSL to the existence of LSS, and Kuchemann [4] called them “the sinews and muscles of fluid motion”. The energy-containing nature of LSS, their scaling, and the effects of Reynolds number on these structures have also long been known and documented in textbooks [5]. Despite all this, the analyses of turbulent FSL in the early years were based on stochastic approaches.

This all changed with two seminal discoveries in FSL, which occurred in the 1960s and 1970s. The first was the finding that FSL, which have vorticity distribution containing a maximum (i.e., a velocity distribution with an inflection point), are unstable to small perturbations over a wide range of frequencies [6–8]. This instability is called the Kelvin–Helmholtz instability, with an essentially inviscid mechanism. Interestingly, although the rigorous analysis of instability in FSL did not begin until the 1960s, it had been noticed in gaseous and liquid jets in the second half of the 19th century [9–11]. The second discovery was the existence of coherent LSS in FSL, even at relatively high Reynolds number. For example, Crow and Champagne [12] showed that axisymmetric FSL act as a band-pass amplifier and can generate and support coherent LSS. Brown and Roshko [13] serendipitously discovered the existence of coherent LSS in high-Reynolds-number planar FSL while they were investigating the effect of density in mixing layers. Winant and Browand [14] demonstrated sequential merging of these structures, albeit in a low-speed, low-Reynolds-number FSL.

Another seminal finding in 1970s [15–17] directly connected instability waves (IW) traveling at supersonic phase speed (with respect to the ambient air) and LSS in FSL to the far-field acoustic...
radiation. These initial discoveries were followed by additional findings clearly showing that the dynamics of LSS in FSL dominate important processes such as entrainment, mixing, momentum transport, and noise generation. These findings motivated extensive research activities in the 1970s and 1980s with the purpose of controlling various processes in these flows. The activities aimed both to gain a better understanding of the flow physics [18–21] and to change the behavior of the flows [22]. This last reference provides a detailed review of these early developments.

Tremendous progress was made in the 1970s and 1980s in the use of active flow control exciting flow instabilities [12,22–29]. However, the experimental research focused nearly exclusively on low-speed, low-Reynolds-number flows (e.g., $Re_l < 50,000$ in jets). As the flow speed and Reynolds number increase, so do the background noise and instability frequencies, requiring high-amplitude, high-bandwidth actuation to excite instabilities. These two opposing requirements impose a very significant demand on the mechanical and acoustic actuators that were used in these early works. As a result, there was practically no experimental work in the active control of high-speed and high-Reynolds-number FSL. Among the few exceptions is, for example, Kibens et al. [30], who successfully used high-amplitude pulsed fluidic injection to excite the flapping mode of the exhaust from a full-scale JT8D jet engine (used in heavy lift military transport). The significant increase in the scale (with the commensurate decrease in required excitation frequency) made this work possible, but it clearly demonstrated the applicability of instability theory and active flow control in high-speed and high-Reynolds-number FSL.

We have recently developed a class of plasma actuators, called localized arc filament plasma actuators (LAFPAs), that can provide local thermal perturbations with high amplitude and high frequency for high-speed, high-Reynolds-number flow control [31–33]. The frequency and phase are controlled independently, allowing several of these actuators to be collectively used to excite not only the free shear-layer instability but also secondary phenomena, for example, various azimuthal modes in axisymmetric FSL. We have successfully used these actuators in several high-speed, high-Reynolds-number flows including jets [32,34], shock/boundary-layer interactions [35], and cavity flows [36,37]. Recently, we have also used a different type of plasma actuator, called nanosecond dielectric barrier discharge (NS-DBD) actuators, which also produce localized thermal perturbations but are more suitable for spatially distributed actuation, to excite and control instabilities in flow over airfoils in both static and dynamic stall conditions [38,39].

This paper will provide a brief review of the previous work in exploring LSS and IW, their roles in FSL, and their control. However, the review cannot be all inclusive because many hundreds of papers have been published on the subject. A brief review of actuators with focus on plasma actuators producing thermal perturbations will also be provided. The paper will then focus on our most recent developments in understanding the flow physics and control of high-speed flows, especially free jets. The role of control in providing a better understanding of the underlying flow physics, as well as the many similarities between low-speed and high-speed FSL physics, for example, LSS pairing and the existence of various azimuthal modes in jets, will be highlighted. These similarities should not be surprising because Morkovin [40] and Bradshaw [41] discussed their similarities for bounded shear flows up to Mach 5 and FSL up to Mach 1.5 several decades ago. To prevent a prohibitive length, the paper will not discuss far-field acoustic radiation generated by LSS/IW. A brief discussion of this topic with appropriate references will be provided at the end of Sec. V.A.

II. Instability Waves and Large-Scale Structures in Free Shear Layers

A canonical free shear layer, or a mixing layer, is formed when two parallel streams with velocity $U_1$ and $U_2$ ($U_1 > U_2$) merge downstream of the trailing edge of a splitter plate, as schematically shown in Fig. 1a. In low-speed FSL, the most influential parameters are the normalized velocity difference $R$ and the Reynolds number based on the initial momentum thickness ($Re_l = U_{ave} \theta_0/\nu$). The latter describes the state of the boundary layer leaving the splitter plate. Another influential parameter, which has often been ignored in the literature, is the disturbance environment. $R$ can vary from 0 to 1, where the two limiting values represent a wake flow and a single-stream free shear layer, respectively. $R$ is a measure of the growth rate of the mixing layer, and $U_{ave}$ is a measure of the convective velocity of the LSS developed in the shear layer. The spreading rate of the mixing layer, $d\theta/dx$, linearly increases with $R$ in incompressible flows, as was demonstrated using heuristic arguments and verified using experimental results by Brown and Roshko [13].

One of the salient features of a mixing layer is that the vorticity profile has a maximum (i.e., the velocity profile has an inflection point). At sufficiently high Reynolds numbers, such flows are known to be unstable to perturbations over a large range of frequencies [6–8,42,43]. This instability is called the Kelvin–Helmholtz instability. Therefore, the third significant (though often overlooked) parameter influencing the development of low-speed FSL is the disturbance environment. If/when the disturbances couple to the flow, they are amplified and generate IW, which grow exponentially in the streamwise direction and roll up into vortices. If the growth of

![Fig. 1 Representations of a) schematic of a FSL, b) variation of the narrowband spectra of the velocity fluctuations at the excitation frequency $f_e$ and its subharmonic $f_e/2$ with $x/\lambda_e$, and c) the momentum thickness variations with $x/\lambda_e$. Adapted from [22].](image-url)
the mixing layer is neglected (assuming a nondiverging shear layer), as was done in most of the earlier investigations, and the basic flow has a profile \( U(y) \), the stream function for the small two-dimensional disturbance growth can be written as [22]

\[
\psi(x, y, t) = \phi(y)e^{i(\alpha x - \omega t)} + \text{c.c.}
\]  

(1)

where \( \omega = 2\pi f \) is the circular frequency of the perturbation under consideration, \( \alpha = \alpha_r + i\alpha_i \) is the complex streamwise wave number, and c.c. represents the complex conjugate. For sufficiently high-Reynolds-number cases, the viscous effects can be ignored, and the linear Rayleigh equation can be solved to obtain the eigenfunction \( \phi(y) \) in Eq. (1) [42]. Researchers have often used a hyperbolic-tangent-type velocity profile, \( U(y, R) = U_{ave}[1 + R tanh(y/2R)] \), or an experimentally or computationally obtained velocity profile for stability analysis. For example, both Betchov and Szewczyk [42] and Michalke [8] used hyperbolic-tangent-type velocity profiles, but the former solved the Orr–Sommerfeld equation, which includes viscous effects, and the latter the Rayleigh equation, which neglects viscous effects. Betchov and Szewczyk’s results [42] clearly describe the effects of Reynolds number on the range of perturbation frequencies to which the shear layer is receptive. They showed that, for \( Re_R > 50 \), the calculated amplification rates are insensitive to Reynolds number, hence validating the use of the Rayleigh equation for this analysis and confirming the inviscid nature of the instability of FSL at high Reynolds numbers. They also showed that, for a given Reynolds number, there exists a most-amplified frequency (wave number) \( f_s(\alpha) \), and the amplification rate decreases as the perturbation frequency deviates from this most-amplified frequency.

The majority of linear instability analyses assume nondiverging shear layers. Although this is obviously an approximation, the results compare surprisingly well with experimental results in terms of the initial amplification of the IW and the development of LSS. Some researchers have used experimental mean velocity profiles (to include the slow growth of the shear layer) and Wentzel–Kramers–Brillouin–Jeffreys (WKBJ) analysis (an approximate solution to linear differential equations with spatially varying coefficients) for their linear stability analysis [44]. The results show excellent agreement between the calculated and measured LSS wavelengths, but only reasonable agreement between the amplitude gains, because the calculated disturbances result in their maximum speed greater than their experimental counterparts. In a series of papers, Goldstein and Hultgren [45], Goldstein and Leib [46], and Hultgren [47] used matched asymptotic expansions in excited FSL, which included weakly nonparallel mean flow. They showed that the initially linear finite growth rate wave was followed by nonlinear effects later in the shear layer. They also showed that including the nonlinearity resulted in significantly farther upstream saturation of the IW [46]. Their results agreed much better [47] with experimental results, highlighting the importance of nonlinear growth mechanisms in FSL [48,49]. A recent work by Cheung and Lele [50] used a combination of simulations and linear/nonlinear analyses of a compressible FSL to show the importance of the effect of the divergence of the shear layer on nonlinear modal interactions and thus on accurate prediction of the saturation amplitude of the IW.

Michalke conducted both temporal [6] and spatial [7] analyses of a single-stream (\( R = 1 \)) FSL. In the spatial stability analysis, the circular frequency \( \omega \) is specified and real, and the wave number \( \alpha \) is unknown and complex. Huerre and Monkewitz [51] use local absolute instability (perturbations spread both upstream and downstream) and local convective instability (perturbations are swept away from the source) to argue in favor of the use of spatial stability analysis in certain flows (e.g., a homogenous mixing layers or a jet with insignificant density variations). On the other hand, they suggest using complex frequency and wave number in certain other flows (e.g., bluff-body wakes and heated jets) because they may exhibit absolute instability. Various instability analyses [7,48] and experimental results [48,52–54] have shown that the normalized spatial growth rate \( -\alpha/\theta \) and phase velocity \( c_s/U_{ave} = \alpha/\alpha_r U_{ave} \) vary with \( St = \theta U_{ave}/U_{ave} \) for all \( R \), and the growth rate has a maximum around \( St_n \approx 0.032 \) (the natural most-amplified frequency of the shear layer). The spatial growth rate increases linearly with \( R \), but the variation between \( R = 0 \) and \( R = 1 \) is only 5%. The normalized phase speed \( c_s/U_{ave} \) is zero for \( St = 0 \), and as \( St \) number decreases to 0 [22]. Therefore, the spatial waves are dispersed below \( St_n \) and nondispersed above \( St_n \) in FSL. Although \( St_n \approx 0.032 \) shows the maximum amplification rate, some researchers have used the maximum amplification instead [26], which occurs around \( St_n \approx 0.024 \) [55] and can only be determined by nonlinear analysis.

All the evidence points to the fact that the growth of the IW and their roll-up into LSS is inviscid in nature at sufficiently high Reynolds numbers. However, the boundary layer on the splitter plate is laminar in low-Reynolds-number flows; thus, the initial mixing layer is laminar and eventually transitions to turbulence. As the Reynolds number increases and the boundary layer on the splitter plate becomes more fully turbulent, the transition location of the mixing layer continues to shift farther upstream. The development of the natural shear layers depends on the nature of the disturbances in a given facility, and the Reynolds number alone cannot capture this effect. Therefore, there are significant discrepancies in the literature on mixing-layer transition location. The reported values of the local Reynolds numbers at the transition location vary from \( 1.6 \times 10^5 \) to \( 6.8 \times 10^5 \) [52,56]. With the turbulent boundary layer on the splitter plate, the reported most-amplified frequency increases by 40 to 50% [57]. The experimental results [13,58] clearly show that the large-scale coherent structures develop over a large range of Reynolds numbers, but the level of small-scale structures embedded in the LSS increases with Reynolds number. This reduces their coherency and two-dimensionality and makes their identification more challenging in experimental work. Wygnanski et al. [59] introduced various strong external perturbations to evaluate the perseverance of the quasi-two-dimensional vortices and showed their existence farther downstream in the mixing layer, even with the presence of such high levels of background disturbances.

The works discussed previously focus primarily on low-speed incompressible flows where density change due to pressure variation is negligible. Mach number, which is the ratio of flow speed to the local speed of sound, is used to indicate compressibility level in a flow and the level of density variation with pressure. For supersonic flows (\( M > 1 \)), shock and expansion waves appear if there is any abrupt change in the flow direction or conditions. In FSL involving two fluids of different velocities and densities, convective Mach number (\( M_c \)) is used as a measure of compressibility. Other measures of compressibility level in various turbulent flows are discussed in Lele [60]. The concept of convective Mach number was first introduced in the numerical work of Bogdanoff [61] and later expanded upon, articulated, and verified experimentally by Papanoschou and Rossho [62]. Convective Mach number represents the normalized convective velocity of large-scale turbulent structures in the shear layer with respect to either the fast- or slow-speed streams in Fig. 1. In this context, a large-scale structure is modeled as a coherent vortex that spans the entire shear layer. For two pressure-matched, parallel streams with equal specific-heat rates (i.e., there are no expansion or compression waves at the splitter plate trailing edge in Fig. 1), the convective Mach number and convective velocity are given as (Papanoschou and Rossho [62])

\[
M_c = \frac{U_1 - U_2}{a_1 + a_2} = \frac{U_1 - U_{c,i}}{a_1} = \frac{U_{c,i} - U_2}{a_2} \quad (2)
\]

\[
U_{c,i} = a_1 U_1 + a_2 U_2 \quad \frac{a_1 + a_2}{a_1} \quad (3)
\]

where \( U_1 \) and \( U_2 \) are the high- and low-speed freestream velocities, \( a_1 \) and \( a_2 \) are the speeds of sound, and \( U_{c,i} \) is the convective velocity. There is relatively good agreement between the measured convective velocity and the theoretical prediction using Eq. (3) for \( M_c \) less than \approx 1, and the scatter in the experimental results is attributed...
primarily to the techniques used and the spatial locations chosen for the measurements. It has been shown by several groups that $M_e$ is effective in characterizing the growth rate of compressible planar FSL, with respect to their incompressible counterparts. The results show a significant decrease in the growth rate of the shear layer as $M_e$ increases. There is, however, significant scatter in the results, which has been attributed to incorrect inclusion of the density effect in extreme density differences between the two streams. The results also show a significant decrease in turbulent fluctuations measured in the FSL as $M_e$ increases. Linear stability analyses results in compressible FSL agree well with the experimental results, showing a significant decrease in the perturbation growth rate as $M_e$ increases. Sandham and Reynolds show that, beyond $M_e$ of around 0.6, the IW become more oblique, and beyond $M_e$ of 1.0, the oblique IW are dominant. The excitation frequency $f_e$ is established. This enhances the growth of the IW and the edge of the splitter plate (the most receptive location for excitation) interacting with the development of the IW/shear layer at the trailing edge of the splitter plate (the most receptive location for excitation) [76]. Under certain conditions, a feedback loop between the downstream traveling IW/LSS and the upstream traveling acoustic waves is established. This enhances the growth of the IW and strengthens the interaction of the IW and shock waves, generating strong acoustic tones called screech tones. Readers interested in the subject are referred to [74,75].

The nature of the flowfield changes significantly when there is a pressure mismatch at the splitter plate trailing edge. This leads to formation of shock waves and expansion waves. The shock waves reflect as expansion waves, and the expansion waves reflect as shock waves from the sonic line within the FSL (because the subsonic regions cannot support these waves) and interact with the IW and LSS, thereby changing their growth and development. The acoustic waves generated by such interactions travel in many directions, including upstream through the subsonic region of the shear layer, interacting with the development of the IW/shear layer at the trailing edge of the splitter plate (the most receptive location for excitation) [76]. Under certain conditions, a feedback loop between the downstream traveling IW/LSS and the upstream traveling acoustic waves is established. This enhances the growth of the IW and strengthens the interaction of the IW and shock waves, generating strong acoustic tones called screech tones. Readers interested in the subject are referred to [74,75].

### III. Instability Waves and Large-Scale Structures in Excited Free Shear Layers

Instability analyses over the past several decades have provided a wealth of information on the physics of FSL. However, there is often confusion in relating IW in the Fourier space to LSS in the physical space in experiments and applications. A linear stability analysis, a mean flow profile, such as a hyperbolic tangent, is often selected. Then, a single frequency perturbation is superimposed on the mean flow, and its growth and decay are calculated. The procedure is repeated for a range of perturbation frequencies. In an experiment, the naturally generated perturbations are generally broadband in both frequency and amplitude because there are many potential resonators (e.g., flow straighteners/turbulence suppressors) and other disturbance-generating sections and parts in the facility (e.g., the contraction from the stagnation chamber to the test section, control valves, bends in the pipes from the air source to the stagnation chamber, steps, etc.). In addition, in high-speed, high-Reynolds-number experiments, the boundary layer on the splitter plate is turbulent and thus contains broadband perturbations. Consequently, the frequency content and amplitude distribution of natural disturbances are wideband and quite different from one facility to another. Therefore, there is always competition among these perturbations, and the perturbations with an amplitude above a certain threshold and frequency within the unstable range of the shear layer are amplified and eventually roll up and grow into the LSS observed in an experiment. This is, for example, one of the reasons for the large variations in the local Reynolds number (from $1.6 \times 10^4$ to $6.8 \times 10^4$) for transition from laminar to turbulent flow in experimental mixing layers in the literature.

In light of the preceding discussion, experiments in low-speed FSL, with a well-controlled disturbance environment, and single-frequency excitation have been instrumental in better understanding the relationship between the IW and flow structures. Figure 1, taken from Ho and Huere [22], is based on the experimental work of Ho and Huang [54] and presents some basic information on this topic. The experiments are done in water with velocities $U_1 = 9.5$ cm/s and $U_2 = 5.0$ cm/s ($R = 0.31$), and the velocity perturbations are created using butterfly valves in one of the waterlines. Figure 1a shows a schematic of the splitter plate and the initial wave, which eventually rolls up into a large-scale structure at $x/\lambda_e$ of around 3, where $\lambda_e = \lambda_e$ is the excitation wave length, and $f_e = \omega_e / 2\pi = f_e$ is the excitation frequency. $\lambda_e$ and $f_e$ are the wavelength and frequency of the wave generated in response to the excitation. Figure 1b shows the variation of the narrowband spectra of the velocity fluctuations with $x/\lambda_e$ at the excitation frequency $f_e$ and its subharmonic $f_e/2$. The spectra are integrated across the shear layer and normalized by $2U_{ave}/\theta_0$. Figure 1c shows the momentum thickness variations with $x/\lambda_e$.

The excitation frequency $f_e$ used to generate Figure 1 is bounded by the most-amplified frequency of the shear layer and its subharmonic, $f_e/2 < f_e < f_e$. Within that range, the shear-layer response is the same as the excitation frequency ($f_e = f_e$). This is often called the “lock-on excitation case”, in which the response frequency is the same as the control frequency [85]. The perturbation grows exponentially and reaches maximum amplitude (saturates) around $x/\lambda_e = 3$, as can be seen in Fig. 1. At that location, the wave rolls up into a structure, and the structure convects downstream as its energy gradually decays. This is also where the linear wave processes end and nonlinear processes begin. While the fundamental wave is growing in amplitude, a subharmonic develops and grows almost linearly, with an energy level nearly three orders of magnitude below the fundamental. The subharmonic briefly saturates when the fundamental wave reaches its maximum growth. Further downstream, the subharmonic begins growing exponentially when the phase speed of the subharmonic matches that of the fundamental (not shown in the figure) [54]. The subharmonic saturates at around $x/\lambda_e = 12$, and at that location, adjacent vortices are vertically aligned, and the process of pairing takes place (also not shown in the figure). If the excitation frequency is close to $f_e$, as was in this case, the pairing of vortices is discouraged and delayed by the overall mean flow. The momentum thickness of the shear layer increases nearly linearly during the amplification stage of the fundamental frequency, saturates when both the fundamental and subharmonic waves are saturated around $\theta_0 \sim 0.075$ [86,87], and grows much more quickly during the period of exponential subharmonic growth (see Fig. 1c). Although not shown here, the momentum thickness normally grows much faster, the wavelength (distance between two LSS) doubles, and the passage frequency halves during each pairing [25]. There have been significant discussions in the literature on the contribution of LSS to entrainment and mixing during roll-up and pairing processes [22].

When the excitation frequency is close to the most-amplified frequency, as in the results shown in Fig. 1, the required perturbation amplitude for excitation, normalized by the freestream velocity, is relatively small. For example, the values used in [48,88] were $10^{-4}$ and $10^{-3}$, respectively. It has been shown that significant increases in the perturbation amplitude (e.g., from 70 to 100 dB) do not change the maximum amplification magnitude but shift its streamwise location upstream [48]. However, as the excitation frequency moves farther away from the most-amplified frequency, the required excitation amplitude increases significantly. The reported non-dimensional perturbation velocity fluctuations values are $10^{-2}$ [12,89] and $10^{-3}$ [20]. Ho and Huang [54] used much lower excitation frequency, $1/3 f_e < f_e < 1/2 f_e$, and showed that the response frequency $f_r$ jumps to the first excitation harmonic $2 f_e$ close to $f_e$. Therefore, the excitation frequency becomes the first subharmonic of $f_e$, and the generated vortices line up vertically and go through pairing. In this situation, as opposed to the case discussed previously, vortex pairing is encouraged and occurs more quickly. Further reduction in $f_e$ causes successive frequency-locking stages,
and \( f_e \) becomes the second and third subharmonics of \( f_\theta \), leading to multiple-eddy or collective merging [54,89].

IV. Development of Large-Scale Structures in Free Shear Layers

As discussed previously, the Kelvin–Helmholtz instability amplifies the perturbations in FSL over a large range of frequencies, and the amplified IW roll up into LSS, as schematically shown in Fig. 1a. These quasi-two-dimensional spanwise structures are shown to be coherent, even at high Reynolds numbers [13]. Dimotakis and Brown [90] showed their existence at a Reynolds number as high as \( 3 \times 10^6 \), and Wygnanski et al. [59] showed their persistence even when various high-amplitude external perturbations are imposed. Therefore, regardless of the initial laminar or turbulent boundary layers, the FSL follows a similar dynamic, namely, the perturbations are amplified and roll up into LSS, and the shear layer eventually becomes turbulent when the local Reynolds number exceeds \( 1.6 \times 10^3 \) to \( 6.8 \times 10^3 \) [56,84]. As the Reynolds number increases, the range of small-scale structures embedded in the LSS increases [13]. Although the spanwise vortices are largely two-dimensional in the spanwise direction, their internal instability eventually leads to the formation of counter-rotating streamwise vortices (distributed in the spanwise direction) with a mean spanwise spacing of approximately \( 0.67 \lambda \), where \( \lambda \) is the streamwise distance between the local adjacent spanwise vortices [58]. The development and dynamics of these structures were studied extensively in the 1970s and 1980s via experimental work [13], vortex dynamics [91,92], and numerical simulations [93]. However, because these structures are produced by an instability, their existence and nature depend strongly upon the perturbation environment, and they are considered a collection of spatially growing IW of various frequencies [51]. For this reason, stability analyses of FSL [7,8] and experimental work using well controlled excitation [54] have provided a wealth of information on the development and dynamics of LSS.

After the fundamental wave with frequency \( f_\theta \) saturates and rolls up into a structure, as shown in Fig. 1a, the successively generated structures convect downstream for some variable distance, depending upon the perturbation environment and whether the pairing is discouraged or encouraged. In the former, they form an array of spanwise vortices, where vorticity is concentrated in their cores, and the vortices are connected to each other via braids and separated from each other by approximately \( \lambda_n = U_{ave}/f_n \) [94]. This scenario could happen in a naturally developed FSL, as in the case of Roberts et al. [94], or when artificial excitation is introduced at a frequency near the most-amplified frequency, \( 1/2f_n < f_\theta \leq f_n \), as discussed in relation to Fig. 1 [54]. On the other hand, if the excitation frequency is near the subharmonic of the most-amplified frequency, \( 1/3f_n < f_\theta \leq 1/2f_\theta \), pairing of adjacent structures is encouraged. Therefore, the successive structures line up laterally and undergo pairing, soon after the initial waves roll up into structures [14,54].

After each pairing process, the structures become significantly larger, their wavelength doubles, and their passage frequency halves [14,25,54]. During the pairing process, entrainment of freestream fluid from both streams is significantly increased, but because the pairing location is random in a natural FSL, the time-averaged momentum thickness grows linearly [22]. As will be discussed later, pairing is also a source of radiated noise [25].

It has long been known that the level of small-scale structures embedded in LSS increases as the Reynolds number increases [13]. This makes it challenging to visualize and identify LSS in compressible mixing layers. However, the limited set of available experimental results clearly shows that the LSS become less coherent and more three-dimensional as \( M_e \) is increased [67,95]. This is also reflected in spanwise correlation measurements, which show reduced spanwise coherence as \( M_e \) increases [96]. Although stability analyses show amplification of oblique IW at higher convective Mach numbers [74,75], experimental results have not yet confirmed these findings [67,70,95–97]. As will be discussed in Sec. VII, we have carried out extensive research using excitation of FSL instabilities in jets over a large range of Mach numbers (up to 1.65), convective Mach numbers (up to 1), and Reynolds numbers based on jet diameter (up to 1.65 \( \times 10^5 \)). The overall nature of LSS, shear-layer growth rate, and generation of Mach waves all depends on the jet Mach number and compressibility level. However, the results clearly demonstrate the similarity of instability processes and the development of LSS in FSL, regardless of the Reynolds number and Mach number. The work included a painstakingly challenging investigation at a Mach 0.9 (\( M_e = 0.45 \)) jet to capture pairing processes, but pairing is not expected to take place at significantly higher convective Mach numbers. However, even at relatively high convective Mach numbers (\( M_e = 1.0 \)), the shear layer still responded to excitation and generated LSS, similar to low-speed flows.

V. Development of Large-Scale Structures in Free Jets

A. Shear Layer and Jet Column Modes

Free shear layers are present in many flows, including free jets, cavity flows, and separated flows. In addition to the momentum thickness \( \theta \), these flows typically have a second length scale, for example, the jet nozzle exit diameter, the cavity length, and the length of the separation region. This length scale establishes another dominant frequency in the flow, which is generally an order of magnitude lower than the most-amplified shear-layer frequency, \( \bar{S}_n = f_\theta \theta/U_{ave} \sim 0.032 \). This second frequency has different names in different applications, it scales with the second length scale, and it is associated with the passage frequency of the LSS farther downstream in the shear layer. This frequency is called, for example, the preferred-mode frequency in jets [12], the Rossiter modes in cavity flows [98], and the shedding frequency in separated flows [99]. Because the goal of this paper is to describe the properties of FSL and the manner in which these can be leveraged to explain the effects of small perturbation-based control, it is necessary to focus the paper appropriately. We thus restrict attention to planar and axisymmetric FSL, which display the main features of interest. Nonaxisymmetric jets [100] as well as multistream jets [101] include additional considerations and phenomena but are not discussed here for brevity.

A streamwise–radial cross section of a jet is shown schematically in Fig. 2. In an axisymmetric jet, the mixing layer has a ring shape.

![Fig. 2 Schematic of a free jet showing roll-up of vortices, the jet mixing layer, and the jet potential core.](image-url)
It begins to develop similarly to FSL, shown schematically in Fig. 1a, but as the ring size and thickness increase with the streamwise direction, the outer diameter increases, and the inner diameter decreases and eventually becomes zero. This occurs when the mixing layer closes itself at the jet centerline, and this location is called the end of the jet potential core. The jet potential core length changes dynamically with time, but its time-averaged length scales with the nozzle exit diameter and is on the order of $5D$ to $6D$, depending on the jet Reynolds number, perturbation environment, and Mach number (among other parameters). The overall developments of the initial IW and the ensuing LSS do not change appreciably regardless of whether the boundary layer at the nozzle exit is laminar or turbulent. It has been shown in the literature that the passage frequency of LSS at the end of the potential core, called the jet preferred or column mode Strouhal number, $S_{p}$, measured experimentally using various techniques, has been shown in the literature to vary from 0.2 to 0.6 [12, 22, 55]. The reason for this variation will be discussed later [83, 102]. The value used in the literature is often 0.3, and this value will be used in the general discussion of this paper.

The initial shear-layer thickness is in some cases much smaller than the nozzle diameter. Therefore, the azimuthal curvature effect can be neglected, and the initial shear-layer development is quite similar to the development of two-dimensional FSL discussed earlier [55, 103] and has been analyzed using linear stability theory [43, 103–106]. However, there are two major differences between the initial development of the IW in two-dimensional FSL and jets. One is the existence of azimuthal modes in axisymmetric jets, which will be discussed later. Another is the limited length of the potential core in jets, which scales with the nozzle exit diameter. After the amplification and roll-up of the IW into LSS in two-dimensional FSL, the further development of the LSS heavily depends on the nature of the disturbances in the flow, whether naturally or artificially imposed. In low-speed and low-Reynolds-number flows, the structures may continue convecting downstream for a long time (if pairing is discouraged) [54, 94] or quickly begin the process of pairing (if pairing is encouraged) [14, 54]. In the right disturbance environment, multiple structures may even amalgamate simultaneously [54, 99] to become one. The amplification and roll-up of the IW into LSS in jets is initially similar to those in two-dimensional FSL, and the frequency of these structures is near the initial shear-layer frequency $f_p$. However, as noted previously, the passage frequency of the structures around the end of potential core is $f_p$, which is an order of magnitude smaller than $f_p$. Therefore, either multiple pairings must take place after the structure roll-up and before the end of potential core or the shear layer must grow sufficiently fast for $f_D$ to approach $f_p D$. Consequently, the initial instability wave development and roll-up, and the potential pairing events, must occur (if at all) between the nozzle exit and the end of the potential core. The structures normally observed in experiments are those with passage frequency of $f_p$, unless special effort is used to visualize the early part of the shear-layer development [107]. It has been shown experimentally that the pairing processes that take place in FSL [14, 54] occur in jets as well and are responsible for decreasing the passage frequency of the LSS from the initial shear-layer value of $S_{p} \approx f_p \theta / U_j \approx 0.016$ to the passage frequency at the end of the potential core, $S_{p} \approx f_p D / U_j \approx 0.3$ [25]. The frequency is halved, and the spacing between the two adjacent structures is doubled by each pairing [25]. Zaman and Hussain [26] noticed that two distinct excitation Strouhal numbers promote pairing in an axisymmetric jet, regardless of whether the boundary layer at the nozzle exit was laminar or turbulent; one scales with the initial shear-layer thickness $S_{p} \approx 0.012$ and promotes pairing close to the nozzle, whereas the other scales with the nozzle exit diameter $S_{p} \approx 0.85$ and promotes pairing farther downstream, $x / D \sim 1.75$.

Michalke [103] provides detailed linear instability analysis of nondiverging and slowly diverging jets as well as the effects of various important parameters such as Reynolds number, Mach number, and temperature on the growth of perturbations. Crighton and Gaster [44] used experimental velocity profiles and WKBJ approximation and obtained a preferred mode Strouhal number of 0.38, which is within the range of experimentally measured values. Petersen and Samet [108] used linear stability analysis with experimentally measured velocity profiles to show that the natural instability in a jet scales with the local shear-layer thickness, which suggests that the preferred mode is actually a shear-layer mode. This is consistent with the assertion of Hussain [55] that the jet shear layer contains two instability modes: the shear-layer mode and the jet column mode. Huerre and Monkewitz [51] called it a slightly damped global mode.

In recent years, researchers have used the parabolic stability equations (PSE) to investigate instability in convectively unstable flows such as boundary layers and FSL. Herbert and Bertolotti [109] developed these nonlinear equations for flows with slowly changing properties in the streamwise direction. Their parabolic nature simplifies the solution procedure and makes them very attractive for linear or nonlinear stability analyses in boundary layers and FSL. Bertolotti et al. [110] used PSE for linear and nonlinear analysis of laminar, incompressible boundary layers. Bertolotti and Herbert [111] used PSE for linear stability analysis of compressible boundary layers. Cheung and Lele [50] used PSE for linear and nonlinear instability analysis in laminar and transitional FSL. Ray et al. [112] used PSE in linear stability analysis of turbulent jets. For the base velocity profile, Cheung and Lele used laminar base flow, and Ray et al. obtained it using Reynolds-averaged Navier–Stokes equations. Guidamundsson and Colonius [113] used experimentally obtained mean velocity profiles with PSE to demonstrate that velocity and pressure fluctuations can be considered linear perturbations to the mean values. The most recent work by Garmaud et al. [114] uses linear amplification of axisymmetric perturbations in an incompressible jet in a fully nonparallel framework. Their results show the existence of a pseudoresonance with a preferred amplification Strouhal number of 0.45. The computationally obtained Strouhal number did not change whether they modeled the external forcing as an inflow condition or a body force.

Following the identification of large-scale coherent structures in turbulent shear layers, stochastic instability wave models were developed to relate the development of LSS to the acoustic wave radiated to the far field. Although the focus of this paper is on flow instabilities and structures, we briefly mention seminal work beginning in the 1970s by Tam [15–17] by Tam and Chen [115] and later by Plaschko [116] relating IW with supersonic phase speed (with respect to the ambient air) to the far-field acoustic radiation. It was shown that, for jets with a supersonic convective Mach number, a “wavy wall” of coherent structures would directly radiate to the acoustic far field on account of their supersonic phase velocity with respect to the ambient fluid. In the case of subsonically convecting structures, the mechanism is more subtle. Modulation of the instability waveform (in the form of amplification or decay) as it convects through the shear layer is necessary to shift energy to supersonic phase velocities and hence produce acoustic radiation [81].

Further experimental studies demonstrated that the peak noise in the acoustic far field is primarily generated by the large-scale coherent structures [117–119]. Cavaliere et al. [120] decomposed the flowfield of a subsonic jet into frequency and azimuthal modes and found good agreement between the measured lower azimuthal modes and a superposition of wave packets predicted by linear PSE. Sinha et al. [121] showed that wave-packet models also reproduced the features of LSS in supersonic heated and unheated jets and that these linear wave packets were responsible for the loudest portion of the far-field noise at aft angles. Temporally and spatially modulated wave packets have been shown to reproduce the directivity and intermittency of the acoustic radiation [122]. Reba et al. [123] used wave-packet models for large-scale mixing noise that successfully propagated the coherent part of the measured near-field hydrodynamic pressure to the far field.

B. Azimuthal Modes

Axisymmetric jets often display unstable azimuthal modes, in addition to the shear layer and jet column modes. The principal factor
in determining the existence and growth of azimuthal modes in the potential core region of a jet appears to be the ratio of the nozzle exit diameter to the shear-layer momentum thickness $D/\theta$. Linear inviscid stability analysis [103] and experimental work [27,28,124,125] show that, for large $D/\theta \ll 1$, both axisymmetric ($m = 0$) and the first spinning or helical modes ($m = 1$) are unstable in the jet shear layer. The analyses [27,28,105] also showed that, for a very thin shear layer (or very large $D/\theta$), many azimuthal modes are unstable in the initial shear-layer region, but the number of unstable modes decreases as the shear-layer thickness increases. Mattingly and Chang [104] used various velocity profiles in the jet core region to show the dominance of the axisymmetric mode when the shear layer is thin and the first helical mode when the shear-layer thickness grows farther downstream. Stromberg et al. [126] showed that the initial part of a Mach 0.9 jet with a low Reynolds number ($Re \approx 3600$) was dominated by LSS of Strouhal number 0.44 and azimuthal modes of $m = 0$ and $\pm 1$. Morrison and McLaughlin [127] used low-Reynolds-number ($Re \approx 8000$) supersonic jets ranging from Mach 1.4 to 2.5 and showed that the dominant instability peak decreases in Strouhal number so that the Helmholtz number $(fD/\alpha)$, where $\alpha$ is the speed of sound at jet temperature) remains approximately constant around 0.4.

Michalet [103] provides a detailed analysis of nondiverging and slowly diverging jets as well as the effects of various important parameters on the growth of disturbances. For an incompressible jet with a plug flow ($\theta_0 = 0$), inviscid linear stability analysis predicts higher growth rate of low-frequency perturbations for the first helical mode ($m = 1$) than for the axisymmetric mode ($m = 0$) and nearly equal growth for higher-frequency perturbations. For larger frequencies $(f \geq f_0)$, the growth of the axisymmetric mode does not depend on $D/\theta$, an indication that $\theta$ is the dominant characteristic length scale. For lower frequencies, however, the disturbance growth rate depends on $D/\theta$, especially for $D/\theta < 100$, where the disturbance wavelength becomes large and comparable to $D$. More recently, Ray et al. [112] used PSE on the mean-flow profile obtained solving Reynolds-averaged Navier–Stokes equations to explore the evolution of IW in transonic jets. They showed that, in the potential core region, $m = 1$ has larger growth than $m = 0$ or 2 at low frequencies. However, at higher frequencies, this disparity diminishes. Most of the experimental work on the excitation of instabilities in jets excited the shear-layer mode [128], the preferred mode [12,25], or the azimuthal modes [27,28,124]. In contrast, Reynolds et al. [129] used simultaneous excitation of the jet column mode ($S_1 \approx 0.55$) and pseudoazimuthal mode (by wobbling the nozzle) at different frequencies. They varied the ratio of the two frequencies and showed a remarkably different distribution of LSS and jet flow development from very large spreading of the jet (“blooming”) to splitting of the jet into two or more jets (“bifurcation”).

Modal analyses, which decompose the flowfield and objectively identify spatial and temporal features with different characteristics, are playing increasingly significant roles in providing a better understanding of flow physics as well as constructing reduced-order models for feedback control [130]. Proper orthogonal decomposition (POD) was introduced by Lumley [131] for primarily hot-wire-type measurements (time-resolved measurements in one or several points). The technique was extended by Sirovich [132] for particle image velocimetry (PIV) type measurements (planar snapshot measurements). POD has been used extensively in FSL, especially jets, in identifying various modes and structures [133–136]. In low-speed jets, Jung et al. [135] show that the dominance of the $m = 0$ mode close to the jet exit decreases relatively rapidly, and the peak modes shift from $m = 6$ to $m = 2$ by the end of the potential core. Tinney et al. [136] show a similar trend of shifting from higher helical modes to lower ones in the streamwise direction in a Mach 0.85 jet. Additional decomposition methods have been proposed to study turbulent shear layers. For example, dynamic mode decomposition was introduced by Schmid [137] to describe the growth rates (as opposed to total energy) of spatial modes at distinct frequencies.

VI. Actuators

There are several classes of actuators that have been used for active flow control, including acoustic, mechanical, fluidic, and plasma. The first two have been in use for quite a while, but the third has seen several variations over the past two decades. They include, for example, synthetic jets [138], Hartmann resonance-based actuators [139–141], spinning-valve actuator [142], and sweeping jets [143]. Because an extensive review paper on actuators is available [144], this section will be brief and focused on plasma actuators, which are the primary devices used to obtain the results presented in this paper.

Plasma actuators have been used for flow control for over 50 years [145], and Martens et al. [146] performed a very early work using them to excite jet instabilities, which demonstrated a significant effect of excitation on the growth rate and spectral content of the developing shear layer in high-speed, low-Reynolds-number FSL. However, the use of plasma actuators as flow control devices primarily began in earnest in the early 2000s. Various surface and volume-filling plasmas, including dc, ac, RF, microwave, arc, corona, and spark discharges, have been used to modify flows. The primary mechanisms of plasma flow control include electro-hydrodynamic (EHD) and magnetohydrodynamic (MHD) interactions as well as thermal (joule) heating. EHD and MHD interactions involve flow entrainment by collisional momentum transfer from charged species accelerated by Coulomb and Lorentz forces, respectively, to the fluid. For MHD flow control, the primary limitation is sustaining sufficient flow conductivity in high mass flow rate conditions. The main limitation of the use of EHD flow control is generating sufficient ion densities in the cathode sheath (space-charge region) of the discharge. Simple estimates [147] show that a significant EHD effect on the boundary-layer flow can be achieved at flow velocities of up to $U_\infty \approx 100 \text{ m/s}$. This is consistent with experimental results using dielectric barrier discharge (DBD) actuators on flow separation control over an airfoil, demonstrated at $U_\infty$ up to $\sim 50 \text{ m/s}$ [148–150]. It has been shown in recent years that, by using much higher input voltage to the actuator and a much thicker dielectric barrier, DBD actuators can maintain control authority at flow speeds up to Mach 0.4 [151]. There are comprehensive recent review papers on this class of plasma actuators [148,152,153]; therefore, they will not be further discussed in this paper. Leonov et al. [154] suggested that significant high-speed flow control could be realized using the thermal effects of near-surface, high-current, high-temperature arc discharges. They showed that intense, localized, rapid heating produced by plasmas in high-current, pulsed, electric discharges produces compression waves, which have the control authority needed to modify even supersonic flows. Localized arc filament plasma actuators (LAFPAs), introduced in 2004 [31], built and expanded upon the concept of the work of Leonov et al. [154]. These actuators were designed to provide control authority to excite flow instabilities in high-speed, high-Reynolds-number flows and were specifically developed to address the shortcomings of other types of actuators for this purpose, particularly the difficulty of achieving simultaneously high-bandwidth, high-amplitude excitation [32]. In this approach, rapid, near-adiabatic heating in the current filament, which also results in compression of supersonic waves, is capable of exciting instabilities in high-Reynolds-number jets [32,33]. A schematic and a photograph of these actuators are shown in Fig. 3. In the earlier work, the electrodes were flush mounted with the inner surface of the nozzle. However, the plasma was noticeably stretched by the high-speed flow and eventually swept downstream, reducing the effectiveness of actuation [31]. Therefore, the electrode tips and plasma arc are now recessed in a circular groove to improve discharge stability and prevent plasma blow. This groove (demonstrated not to significantly affect control authority [155]) shields the arc filament, allowing it to be sustained or pulsed at any desired frequency to most effectively excite natural instabilities over a wide range of frequencies. The high-current, high-temperature, localized arc discharge is completely different from the low-temperature, low-current, diffuse glow discharges used in previous low-speed flow control work [148]. In this approach, the flow is affected by localized perturbations produced by arc-generated...
temperature spikes followed by compression waves (a purely thermal effect). The present approach is not limited to low-speed flows (as EHD control) or low-pressure flows (as MHD control) and requires low enough energy, as it targets natural instabilities in the flow, to be used in various applications.

There are two variations of pin-electrode-based plasma actuators besides the LAFPAs, namely spark jet [156,157], and pulsed plasma jet [158]. In the first, the electrodes are located in a small cavity and can be used to generate a pulsed high-speed control jet. In the second, the electrodes are located at the throat of a cavity with converging or converging–diverging walls to modulate the sonic flow at the throat. Although this last is a more recent development, the other two actuators are high-amplitude actuators and have been successfully used in several applications. However, their bandwidth is limited due to the existence of the cavity. Adelgren et al. [159] used only a single pair of electrodes located at the nozzle exit to control a high-speed jet.

The concept and initial development of LAFPAs are presented in [31]. Later evolution and characterization of LAFPAs, as well as the differences between LAFPAs and other plasma based actuators, are given in [32,33]. In its basic form, a LAFPA consists of a pair of electrodes, one connected to ground and the other to a high-voltage (several kilovolts) source. In typical laboratory experiments, the actuators are located approximately 1 mm upstream of the shear-layer origin, and the center-to-center distance between the two electrodes is typically 3–4 mm. High-voltage pulses are applied to each actuator by means of an electronically controlled switch. The high initial voltage is needed to achieve breakdown in the approximately atmospheric pressure air in the (3–4 mm) gap between the electrodes. When the voltage across a pair of electrodes reaches the breakdown voltage, the air between the electrodes breaks down, and an electric arc is established. After the breakdown, the voltage across the electrodes drops to a few hundred volts and remains at that level until the voltage source is disconnected (Fig. 4). The frequency and duty cycle/pulse width are controlled independently for each actuator, and limits (determined by the power supply) are typically tens to hundreds of kilo Hertz and 1 μs to 1 ms. Past results have demonstrated that complete breakdown of the air is crucial to achieve significant control authority [34,155].

Moore [20] states that the shear layers are receptive to thermal, acoustic, and hydrodynamic perturbations, provided that the perturbations are spanwise coherent. Eight LAFPAs are distributed uniformly along the nozzle exit perimeter (Fig. 3) to provide the most azimuthally (spanwise) uniform perturbations possible. In addition, the eight actuators can be operated with phase delays and patterns to excite higher azimuthal modes. The actuators are located approximately 1 mm upstream of the nozzle exit, as close to the origin of the shear layer as physically possible, because the origin is the region of greatest receptivity [76]. Kim et al. [160] simulated the full nozzle geometry, incorporated the plasma through a phenomenological volume-heating-based model, and showed reasonable comparison with the experimental...
results. In a high-fidelity computational work, Gaitonde and Samimy [161] used a simple surface heating model, based on the experimentally measured temperature perturbations. This methodology produced mean flow results and LSS with various azimuthal modes, which compare quite well with experimental results, highlighting the dominance of the Joule-heating-based control mechanism.

The short-duration, harsh, high-temperature environment of the plasma makes accurate measurements of perturbations imparted to the flow by the actuators quite challenging. We have used nitrogen emission spectroscopy to measure the average temperature of the plasma, which depends on the frequency and duty cycle of operation. The measured temperature, averaged over the spatial extent of the plasma (typically about 1 mm wide and 3–4 mm long) and over several pulses, varies from a few hundred to about 1200°C [162], depending upon the duty cycle. As was discussed, the jet is known to be receptive to thermal, aerodynamic, and acoustic perturbations [20]. With LAFPAs, the initial perturbation is thermal. However, the flow is compressible, and the rapid microsecond-time-scale localized heating generates an acoustic wave, as shown in Fig. 5. The flow in this figure is a Mach 0.9 jet exhausting from a rectangular jet with an extended lower lip to house the actuators and aid the visualization of the generated waves. The evolution of a thermal input into an acoustic wave has been explored computationally by Gaitonde [163]. It is unclear whether it is the thermal perturbation, the acoustic perturbation, or a combination of the two that is coupled to the flow. As detailed in a computational work [76], the shear layer is receptive both to thermal perturbations and the resultant acoustic perturbations. By positioning the actuators as close as possible to the shear-layer origin, the phase difference between the excitation provided by the thermal and acoustic perturbations in minimized, thereby maximizing the overall effect. However, the thermal perturbation seems to play a significant role as simulations [161] have used it to model the plasma and achieve flow control results that agree quite well with experimental results. The earlier results showed that the actuators could generate streamwise vorticity and vortical/aerodynamic perturbations, the strength of which depends on the distance between the electrodes [31]. However, with only 3 or 4 mm between the electrodes in most current works and potentially nonuniform plasma between the electrodes, the generated streamwise vorticity is expected to be quite weak.

Although the instantaneous current, voltage, and power dissipation are quite high (see Fig. 4), a short pulse-width is used, resulting in very low (O(10) W per actuator) time-averaged power consumption. Specifically, with the conditions used in Fig. 4, the steady-state arcing power was approximately 100 W. Thus, for a 20% duty cycle, each actuator consumes a time-averaged power of approximately 20 W. Samimy et al. [34] confirmed that the extremely rapid heating associated with the breakdown process did indeed provide the perturbation required to excite flow instabilities. Furthermore, Samimy et al. [164] demonstrated that there is a slight improvement in performance with decreasing pulse width, ascribed either to the lack of significant steady-state heating or to the salience of the perturbations, as long as consistent breakdown is maintained. Although the extremely short time scales (order nanoseconds) associated with breakdown give a maximum theoretical excitation frequency in the tens to hundreds of mega Hertz, currently employed power supplies limit these values to O(100) kHz. These values are, however, well within the range required to excite the initial shear-layer instability of even very high-speed FSL.

Because the power supply is the primary limiting factor on the operational envelope of the LAFPAs, the Gas Dynamics and Turbulence Laboratory (GDTL) has invested substantial effort in improving their design. The three most important aspects of the power supply will be briefly highlighted. The first is the excitation frequency. The rapid changes in the current associated with pulsing the actuators cause significant energy dissipation within the power supply. This energy dissipation scales linearly with frequency. Thus, increasing the available bandwidth typically involves improving heat dissipation. Although the frequency response of the driving electronics controls the minimum rise time of the voltage to breakdown levels, effectively limiting the maximum theoretical frequency of the discharge, in practice insufficient heat dissipation is typically the limiting factor. Second, although some level of electromagnetic interference (EMI) will always be generated by the arc, the vast majority of EMI is radiated by the power cables and internal power supply circuitry. This problem poses a significant concern for applications. However, because it can typically be avoided via adequate shielding in a laboratory environment, little consideration has been given to it in the design of the power supplies in our laboratory. Third, the electronic efficiency of the power supply must be improved. The first-generation power supply (Fig. 6a) used high-bandwidth, high-voltage switches to rapidly pulse the actuators, together with large, high-voltage, ballast resistors. Although this design is simple to implement, it is expensive (in particular the high-bandwidth, high-voltage switches) and very inefficient. The second-generation system (Fig. 6b) addressed some of these problems by switching to low-voltage power and using a step-up transformer to achieve the necessary breakdown voltages. This arrangement greatly increases the efficiency of the power supply and allows the control circuitry and switches to be implemented as a printed circuit board, thus reducing the cost, size, and weight of the system. However, the electronic recoil associated with rapidly loading and unloading the transformers causes this design to produce more EMI than the first-generation system. These three requirements involve trades that must be balanced based on the particular application, available power, and desired operational envelope of the LAFPAs.

Although the power supply limits the actuation fundamental frequency, this limitation does not prevent LAFPAs from exciting flows with instabilities that preferentially amplify frequencies within an envelope higher than the LAFPAs’ fundamental frequency. As discussed earlier and shown in the literature using acoustic perturbations, the larger the differences between the actuation and the most-amplified frequencies are, the larger the required perturbation amplitude is [12,20,48,88,89]. The impulsive nature of LAFPA excitation (see Fig. 4), especially because the breakdown provides the bulk of the perturbation, results in the introduction of perturbations not only at the fundamental actuation frequency but also at several of the harmonics as well. This is readily seen in irrotational near-field pressure spectra (Fig. 7) collected from an excited Mach 0.9 jet [107,165]. The high-amplitude nature of the LAFPA-produced perturbations ensures that at least one of the several harmonics has sufficient amplitude to excite natural instabilities. The actuation signal is superimposed on the background pressure signal, modifying it slightly at \( \text{Str} = 0.05 \) and significantly at \( \text{Str} = 0.35 \), which is near the jet column mode Strouhal number. Similar actuator-produced perturbations (i.e., the fundamental frequency along with several harmonics) have been reported by Visbal [166,167] and Visbal and Benton [168] in a large-eddy simulation where periodic blowing and

Fig. 5 Schlieren image depicting actuator-generated compression waves.
suction with a step function profile for flow separation control were employed.

Following the development of LAFPAs, it was reported in Roupassov et al. [169] that if DBD actuators (previously used to impart momentum to the flow) [148] are driven by a nanosecond pulse waveform (rather than the typical AC waveform), they produce thermal perturbations rather than imparting momentum to the flow [170]. These actuators, termed nanosecond DBD (NS-DBD), are similar to LAFPAs and produce relatively high-amplitude, high-bandwidth perturbations for effective instability-based flow control. The primary difference between LAFPAs and NS-DBD actuators is that, although LAFPAs produce spatially localized, discrete perturbations (and are typically used in groups of actuators distributed throughout the receptivity region), the NS-DBD actuators produce spatially distributed perturbations. Recently, NS-DBD actuators have been used to control flow over airfoils in both static and dynamic stall conditions by manipulating the instabilities associated with the flow [38, 39, 170, 171]. At least one paper within this special issue provides significant details on the nature and operation of NS-DBD actuators [172].

Thermal-based actuators for excitation of instabilities (LAFPAs and NS-DBD) have ushered in the most recent developments in flow control. The high-bandwidth, high-amplitude perturbations these devices generate, and their dynamic flexibility, have not only opened the possibility of excitation of natural flow instabilities in previously inaccessible high-speed, high-Reynolds-number flows, but they have also allowed optimizing, dynamic, feedback control in both these and other flows. The flow control community has now been exploring these exciting new areas for nearly two decades, and the rest of this paper provides some sample results in jet flows that LAFPAs have made possible.

VII. Recent Results on Excitation of Instabilities in Free Jets

Experiments exploring the excitation of instabilities in free jets using LAFPAs have been conducted in the anechoic chamber at the GDTL. Details of the facility can be found in Hahn [173]. In brief, it has internal dimensions of 5.14 m wide by 4.48 m long by 2.53 m tall and has a design cutoff frequency of 160 Hz. Compressed air is supplied from storage tanks with a total capacity of 43 m³ and maximum storage pressure of 16 MPa. The air may be routed through a storage heater, which allows the jet to operate with a stagnation temperature up to approximately 600°C (∼1110°F), before expanding through a nozzle and exhausting horizontally into the chamber. The chamber has been designed to facilitate data acquisition with a number of different experimental methods, including hot-wire anemometry, near-field and far-field pressure/ acoustic measurements using microphones, schlieren imaging, particle image velocimetry (PIV), and tomographic PIV.

A range of different nozzle configurations have been studied (rectangular, axisymmetric, twin-jet, contoured, and biconical), though contoured (both converging and converging–diverging) axisymmetric nozzles of 2.54 cm (1 in.) exit diameter have been used most commonly. The nozzles are thick-lipped to facilitate the use of a nozzle extension, which houses the LAFPAs. With a single jet, eight actuators are used, as schematically shown in Fig. 3. The boundary-layer characteristics were examined using a hot wire in an unheated jet. With the actuator extension installed, good collapse of the shear-layer velocity profiles was obtained over a wide range of Reynolds numbers, indicating that the boundary layer is fully turbulent for Reynolds numbers larger than 1.6 × 10⁵ [124]. This range encompasses the entirety of the experiments conducted to date in the GDTL anechoic chamber. The hot-wire data were used to estimate the initial momentum thickness as ∼0.09 mm, with D/θ ∼ 280 and the boundary-layer thickness as ∼1.2 mm at the nozzle exit. This thin shear layer is highly receptive to various azimuthal modes, as discussed earlier, and thus allows the use of LAFPAs for excitation of FSL in jets over a wide range of frequencies covering the shear-layer mode, jet-column mode, and azimuthal modes.

In the following section, selected results will be presented and discussed in three areas: jet response to excitation measured in the irrotational near field, development of LSS in the excited jets, and the effects of excitation on global properties of jets. References will be provided for readers interested in more detailed results and discussion on each of these topics.

A. Jet Response to Excitation Measured in Irrotational Near Field

The harsh environment (i.e., high temperature and electromagnetic interference, EMI) generated by LAFPAs makes flow diagnostics near the nozzle exit, where the actuators are located, very difficult. Therefore, investigation of the response of the jet to excitation of various frequencies has focused on the irrotational near-field pressure measured using a microphone array shown in Fig. 8. The first
microphone is at $x/D = 1, r/D = 1.2$, which is located just outside of the shear layer of the jet. The array is oriented at an 8 deg angle with respect to the jet centerline to accommodate the shear-layer spreading, and so all the microphones are located just outside the jet shear layer. In this essentially irrotational field, they measure the hydrodynamic pressure (i.e., the signature of the LSS within the shear layer), allowing their spatial evolution, wavelength, timing, and nature to be investigated. Sinha et al. [121] explored the effects of LAFPA excitation on axisymmetric jet response by way of time-domain analysis of the instantaneous pressure fluctuations in the irrotational near field of the jet. Triple-decomposition of the instantaneous fluctuations into the mean, a coherent “wave” component and the incoherent fluctuations [175] was accomplished by phase-averaging the signals over a large number of excitation periods. For axisymmetric excitation (that is, all eight actuators operated in phase), the signature was a series of nearly sinusoidal positive and negative fluctuations (Fig. 9a). Because the first microphone is located at $x/D = 1$, the IW produced by the excitation have already rolled up into structures at this location (as will be shown later).

It was found that, at very low Strouhal numbers ($St_{De} < -0.1$), the excitation produced a compact waveform (with limited temporal extent) that had a temporal persistence (i.e., wave period) less than the excitation period by nearly an order of magnitude. In other words, each excitation signal produced a perturbation that was amplified by the shear-layer instability, formed an instability wave, and rolled up into a structure that produced a near-field pressure signature with much higher-frequency content than the actuation frequency itself. The reason for this phenomenon is the existence of higher harmonics of the actuation signal, as discussed in the actuator section and shown in Fig. 7. Therefore, one of the harmonics of the actuation signal, whichever is closest to the most-amplified frequency of the jet shear layer, is amplified and rolled up into a structure, and this signature is detected in the irrotational near field. Thus, the jet responds to the harmonic of the actuation frequency closest to the most-amplified shear-layer frequency. This response is referred to as the “impulse” response to the excitation because it represents the jet response to a single, impulsive perturbation. The characteristic response is constant for excitation periods greater than the temporal persistence of the characteristic response. This waveform was amplified for a short distance, as shown in Fig. 9a, before beginning to decay near the end of the potential core ($x/D = 6$ for Mach 0.9 jet). Figure 9b shows the maximum waveform amplitude as a function of spatial location to more clearly depict the rapid growth and subsequent decay of the response waveform.

As the actuation Strouhal number increases to near the preferred mode Strouhal number, the jet response matches the period of excitation, as demonstrated by Fig. 10a (for a Mach $= 0.9$ jet at $x/D = 3$ and for various excitation Strouhal numbers indicated on the caption). This is termed the “periodic” response of the jet. For excitation frequencies below the jet column mode frequency ($St_{De} < -0.3$), the fundamental shape of the response is still largely unchanged from the impulse response, though at higher frequencies (e.g., $St_{De} = 0.50$), a distinct reduction in the fluctuation amplitudes is apparent. However, it was found that this periodic response could be well approximated by a linear superposition of the impulse response, repeated at the higher excitation frequency with the proper phase shift. This behavior is illustrated in Fig. 10b for $St_{De} = 0.50$. Clearly, the periodically seeded waves are primarily interacting with each other in a linear manner. However, at even higher frequencies ($St_{De} > -0.50$), this quasi-linear model no longer correctly predicts the jet response, suggesting increasingly important nonlinear dynamics.

The measured irrotational near-field pressure is composed of nonradiating (exponentially decaying in the radial direction) hydrodynamic and radiating (propagating) acoustic pressure waves [176]. They can be separated using various techniques, including filtering in the Fourier domain [177] and wavelet analysis [178] or more generally using momentum potential theory [179]. In the results reported in Figs. 9 and 10, the acoustic Mach number $M_a = U_a / a$, where $U_a$ is the jet velocity at the nozzle exit and $a$ is the ambient speed of sound, is subsonic. This is true in subsonic low temperature jets. Therefore, the irrotational pressure just outside of the jet shear layer is dominated by the hydrodynamic component [117]. Because the microphone array is located just outside of the shear layer, filtering the data to remove the acoustic component does not significantly change the results presented in Figs. 9 and 10 [180]. However, as the hydrodynamic pressure waves decay exponentially in the radial direction, the dominance of hydrodynamic pressure waves vanishes at relatively modest distances from the jet. Additional details on the hydrodynamic and acoustic fields, as well as how to decompose and independently analyze these two fields, can be found in Crawley et al. [178,180] and the references therein.

![Fig. 8 Photograph of the anechoic chamber, nozzle with eight actuators, and near-field linear microphone array.](image)

![Fig. 9 Phase-averaged impulse response waveforms: a) measured at different streamwise locations over a single actuation period for $St_{De} = 0.05$, and b) maximum amplitude plotted as a function of streamwise location.](image)
B. Development of Large-Scale Structures in Excited Jets

Further insight into the initial development of LSS in an excited jet is provided by the results of Crawley et al. [107, 165]. In this work, the development of axisymmetric ring vortices within the jet shear layer over the jet core region of a Mach 0.9 jet with a Reynolds number of $6.2 \times 10^5$ was investigated using time-resolved, near-field pressure measurements and simultaneously acquired synchronized non-time-resolved, instantaneous velocity measurements provided by streamwise planar PIV. Data were collected while the jet was undergoing excitation over a range of Strouhal numbers. Using a form of multi-time-delay stochastic estimation, a low-order estimate of the time-resolved velocity field was produced. Figure 11 plots swirling strength [181] in the jet excited near the jet column frequency at $St_{De}/0.01360.35$ at successive phases over one excitation cycle to show the growth, development, and convection of the LSS generated by amplification of perturbations by Kelvin–Helmholtz instability in the jet shear layer. Because the excitation was axisymmetric, only the top half of the jet is shown (i.e., an apparent single vortex is actually a planar slice of an axisymmetric ring vortex).

It is observed that the excitation produces periodic LSS, the wavelength (spacing between two adjacent vortices, $\lambda$) of which roughly matches the expected vortex spacing beyond $x/D > 4$ for this excitation Strouhal number. However, just downstream of the nozzle exit, smaller closely spaced structures are present; in fact, as the structures advect downstream, multiple stages of structure mergings occur before the final passage frequency or wavelength is achieved. Starting from the first (top) frame, two small vortices are discernible just downstream of the nozzle exit ($x/D < \sim 0.5$). In the second frame, they have grown and convected downstream; more importantly, though, the trailing vortex has accelerated slightly relative to the leading vortex and is in the process of merging with the leading vortex. This process continues through frame 3, and by frame 4, there is one coherent large-scale structure in place of the two
initial vortices. As the excitation cycle repeats, this single coherent structure, while advecting downstream, is now located at \(x/D = 2\) in frame 1. Here, it begins to undergo another merging process, accelerating relative to the preceding structure (at \(x/D = 2.6\) and being induced into it. This second merging process is complete by \(x/D = 3.5\) (frame 4). At this point, the structure begins to decay as it passes through the end of the potential core (at \(x/D = 6\) in the baseline jet). Another potential merging process (though of disparately sized structures) may be observed beginning at \(x/D = 1\) in frame 1 and concluding at \(x/D = 2\) in frame 4. Each merging reduces the frequency and increases the wavelength by a factor of 2

Therefore, given the actuation Strouhal number of 0.35, two sequential merging events would suggest that the jet is responding to excitation at a Strouhal number of 1.4 (the third harmonic of the actuation frequency, \(f_1 = 3f_0\)), and three sequential merging events would suggest that the jet is responding to excitation at a Strouhal number of 2.8 (the seventh harmonic of the actuation frequency, \(f_1 = 7f_0\)). Typically, two to three sequential mergings are needed to reduce the frequency from the most-amplified shear layer to the jet preferred-mode frequency in low-Reynolds-number, low-speed, excited jets [25]. These results are similar to the results of Ho and Huang [54] concerning low-frequency excitation in very low-speed and low-Reynolds-number FSL, and to the authors’ best knowledge, this is the highest Mach number and Reynolds number at which merging of LSS in FSL has been observed.

The location and number of these merging events was found to be dictated by the excitation Strouhal number. In the case of impulse excitation, no clear merging events were observed. The generation of structures with a higher spatial frequency and their subsequent merging helps to explain how the near-field pressure contains energy at the fundamental actuation frequency. As discussed previously, and in Sec. VI, LAFPAs introduce a nonsinusoidal perturbation into the shear layer that contains energy at the fundamental actuation frequency as well as higher harmonics. Therefore, the excitation harmonic closest to the most-amplified frequency of the jet shear layer is likely amplified and rolls up into structures, the signatures of which are measured in the irrotational near field.

The annular shear layer of the axisymmetric jet gives rise to an additional instability modal parameter: that of the azimuthal (Fourier) mode. In addition to the excitation frequency, the excitation azimuthal mode has been explored extensively with LAFPAs and shown to strongly affect the LSS dynamics and jet global flowfield. The discrete nature of LAFPAs requires the true Fourier modes to be approximated by their binary equivalents. For a 1-in.-diam jet with eight actuators, the axisymmetric mode \(m = 0\) (ring), the helical modes \(m = 1\) (helix), \(m = 2\) (double-helix), and \(m = 3\) (triple-helix), and the complex modes \(m = \pm 1\) (flapping), \(m = \pm 2\), and \(m = \pm 4\) can be well approximated. The effect of azimuthal excitation mode is demonstrated in Fig. 12 for a Mach 1.3 axisymmetric jet, where phase-averaged streamwise planar Mie-scattering-based visualization results are presented. The laser light is scattered by condensed water droplets in the mixing layer (as a result of mixing cold, dry jet air with the entrained warm, moist ambient air) and highlights the LSS generated by excitation. In all cases (except the baseline, which is an ensemble-averaged image), phase-averaged images (phase-locked to the actuations signal) of the jet excited at \(St_{1D} = 0.33\) have been acquired and displayed. This frequency was chosen because it is near the jet column mode and thus produces the largest amplification of the perturbations and the most robust structures in the downstream region of the jet. The shear layer immediately downstream of the nozzle is not visualized because negligible entrainment and mixing occurs while the IW are growing and rolling up into structures, and thus very few water droplets are present to scatter the laser light in this region.

As expected, given the phase relationship of the excitation modes, excitation of the axisymmetric mode produces structures that are symmetric about the jet centerline, whereas the first helical mode and flapping mode produce structures that look antisymmetric. For all cases shown here, with very large \(D/\theta\) (greater than 250), many azimuthal modes in the jet initial shear layer are unstable [27,28], as the results in Fig. 12 demonstrate. Noticeable differences are apparent between the different azimuthal modes in the structure growth and ultimately in the spreading rate of the mixing layer. Both the axisymmetric and first helical mode exhibit only moderate growth of the shear-layer width, with most of this growth occurring upstream of the end of the potential core. In contrast, flapping mode excitation results in a significant increase in spreading rate of the shear layer in the flapping plane, up to and beyond the end of the potential core.

Although in a planar slice of the large-scale flow structures (as presented in Fig. 12), the LSS produced by helical and flapping mode excitation appear quite similar, this is not the case. Though the flapping mode produces a significant enhancement of the spreading rate in the flapping plane, the spreading is significantly reduced in the nonflapping plane [32]. As detailed in a large-eddy simulation work by Gaitonde and Samimy [161], the structures produced by the flapping mode excitation are a highly complex alternating chain of vortices whose axis lies in the flapping plane and are inclined with respect to the flow direction. As a result, the jet mixing layer becomes elliptical in the downstream region.

The effect of LAFPA excitation on the LSS has been explored over a wide range of jet Mach numbers, Reynolds numbers (varied independently of Mach number by altering the total-temperature ratio), and excitation frequencies and modes [164]. Sample results...
shown in Fig. 13 show the effects of excitation Strouhal number on the development of LSS, where the structures have been identified in phase-locked streamwise velocity fields by overlaying Galilean streamlines [182] with normalized Q-criterion [183]. The combination of these vortex identification methods clearly illuminates the core and braid regions of the coherent LSS. Here, the results are shown for an unheated Mach 1.3 jet, though they are representative of the results for other Mach numbers and temperatures. Remarkable consistency of the results was found when exciting the jet at frequencies near the jet column mode [164]. This consistency indicates the dominant control authority of excitation over the development of the LSS. Increasing the excitation frequency causes a reduction in the structure size as well as the structure spacing or wavelength.

As discussed previously, for moderate Strouhal numbers ($St_{De} < \sim 0.50$), the structures grow, advect, and ultimately decay and disintegrate near the end of the potential core. As the excitation frequency is increased beyond the jet column mode, however, the generated structures are still coherent in the phase-locked images, though they are reduced in size, commensurate with the excitation frequency, and develop and disintegrate much farther upstream. This consistent structure spacing is demonstrated in Fig. 14, where the structure spacing is computed from spatial correlations of the PIV images using either the radial velocity [184] or Q-criterion [185]. The resultant structure spacing collapses well onto a single curve defined only by excitation Strouhal number across all Mach numbers, total-temperature ratios (TTR), and excitation modes investigated. Interestingly, if it were not for the constant (0.327) in the equation representing the fitted curve, the results would indicate a constant convective velocity of LSS ($U_c = 0.51U_j$) over all the Mach numbers and excitation frequencies included in Fig. 14.

The perturbations imparted by the actuators are amplified by the shear-layer instability as IW that roll up into LSS. As the LSS convect downstream, they produce coherent pressure fluctuations that are easily identifiable in the near-field pressure results (see Figs. 9 and 10). These pressure fluctuations are related to the passage of the core and braid regions of the LSS and can also be observed from the density gradients obtained by phase-averaged schlieren images, provided that the Mach number is sufficiently high to generate substantial density gradients. Figure 15 depicts results from a Mach 1.3 jet excited with different combinations of azimuthal mode (axisymmetric or helical) and frequency ($St_{De} = 0.3$ or 0.6) at a TTR of 1.5. Because of the varying TTRs, the acoustic Mach number (jet velocity normalized by ambient speed of sound, $Ma_c = U_j/a$) varies from $Ma_c = 1.1$ (for TTR of 1.0), which is not shown, to 1.4 (for TTR of 1.5). The convective velocity of the LSS has been shown to vary slightly with structure frequency and azimuthal mode. The convective velocity of the LSS has been shown to vary slightly with structure frequency and azimuthal mode, though it is generally $U_c = 0.7U_j$; hence, the convective Mach number of the LSS varies from roughly $Ma_c = 0.8$ to 1.0. It has been shown experimentally that, in natural jets, when $Ma_c$ approaches 1.3, the convection of LSS generates noise, called Mach wave radiation [186], which can be seen in schlieren images in both the near and far field.

As discussed earlier, excitation of instabilities can significantly impact the flow and pressure fields, thus significantly increasing or decreasing the rate at which ambient fluid is entrained into the mixing layer and propagate with the leading edges of the structures. As the acoustic Mach number increases to $Ma_a = 1.4$ ($Ma_a = 1.0$, Fig. 15), these pressure fluctuations coalesce into sharp, high-amplitude wave fronts, Mach waves, which radiate to the far field. It was demonstrated in Kearney-Fischer et al. [187] that the shape of this wave front shifts from bowed to conical as the convective velocity increases, corresponding to the change in the dominant radiation direction. The excitation frequency and mode were found to affect the emergence of these sharp wave fronts; frequencies and azimuthal modes that produced the strongest coherence in the LSS (that is, the axisymmetric mode at frequencies near the jet column mode) also produced the most coherent Mach wave radiation. On the other hand, higher-order modes (such as $m = 3$) and very high frequencies did not produce readily identifiable Mach wave radiation, even in the phase-averaged schlieren images.

Some current advanced tactical aircraft (such as the F-15 and F/A-18) use a twin-jet configuration, in which two exhaust jets are located roughly parallel to one another and at a small relative distance from each other. In such jets, the flow, pressure, and acoustic fields of the two jets can interact and potentially generate much more intense unsteady near-field pressure fluctuations (potentially damaging the aircraft structure) and far-field radiation. Kuo et al. [188] recently used eight LAFPAs on each nozzle to excite the jets’ shear-layers instabilities and azimuthal modes and either couple or decouple the two jets’ flow and pressure fields, thus significantly increasing or decreasing the near-field unsteady pressure fluctuations.

C. Effect of Excitation on Global Properties of Jets

As discussed earlier, excitation of instabilities can significantly change the development, size, and nature of the LSS and thereby change the rate at which ambient fluid is entrained into the mixing layer.
layer. This change in the entrainment has the dual effect of both modifying the spreading rate of the jet shear layer (and thereby jet width) and altering the length of the potential core region. As expected, the particular excitation Strouhal number at which the overall entrainment is maximized is a function of the excitation azimuthal mode, though it typically occurs at a frequency near the jet column mode frequency [184,189].

It is nontrivial to assess the effect of excitation on the global features of the jet, especially because, as different azimuthal modes are excited, planar streamwise PIV measurements on planes with different azimuthal angles provide significantly different results. However, this is not the case for axisymmetric excitation. Figure 16 shows the jet width at half of the jet centerline velocity (δ = 2r₁/₂, r₁/₂ defined as the radial location where velocity is 50% of the centerline velocity) for a Mach 0.9 jet excited with the axisymmetric mode over a large range of Strouhal numbers from below the preferred-mode Strouhal number (Stₚ) to above the most-amplified Strouhal number (Stₙ). The jet potential core ends around x/D = 6 for the baseline jet [184], which shows typical initial linear growth, followed by saturation approaching the end of potential core, followed by another region of linear growth with a sharper slope, starting around x/D = 8. For the controlled cases, as the excitation Strouhal number increases toward the jet preferred mode (Stₚ = 0.3), the maximum growth rate is achieved. The curve now has three different regions, with each having a higher growth rate than the one before. At much higher Strouhal numbers than that of the jet column mode, the curve shape returns to the baseline shape, except the first region of linear growth becomes shorter, and the saturation region becomes longer, covering x/D from 1 to 7.

The jet growth rate results are consistent with the flow structure results shown in Fig. 13. Basically, excitation at frequencies near the jet column mode generates large coherent structures, which can significantly increase the entrainment of ambient air into the jet and increase the jet growth rate or reduce the jet core length. On the other hand, excitation at much higher Strouhal numbers generates smaller, less coherent structures and limits the extent of their streamwise development, thereby significantly decreasing the entrainment and growth rate. Although Fig. 16 shows the results for the axisymmetric excitation mode, the increase in entrainment and mixing is more substantial when using the flapping azimuthal mode (m = ±1), as can be seen in Fig. 12, and less substantial when using higher azimuthal modes (e.g., m = 3).

Although the paper has focused on the fluid dynamics aspects of control, these results have significant implications for using flow control for jet noise mitigation. It has been known for over five decades that the dynamics of LSS generate the highest level of noise at a Strouhal number near that of the jet preferred mode and radiated in the 30 deg direction with respect to the jet centerline [81]. However, the peak noise signal is intermittent and was not well understood until relatively recently [118,119]. Reducing dynamics of LSS as well as their spatial extent by high-Strouhal-number,
high-azimuthal-mode excitation, as shown in Figs. 13 and 16, provides opportunities for significant noise mitigation [164].

VIII. Summary

Free shear layers are present in many practical and canonical flows of interest, including, for example, jets, cavity flows, and separated flows. The following were found in the 1960s and 1970s.

1) FSL are unstable to small perturbations over a wide range of frequencies.
2) Coherent LSS are present in FSL, even at high Reynolds numbers.
3) IW and LSS traveling at supersonic phase speed (with respect to the ambient air) generate intense noise with a preferred far-field radiation direction.
4) The dynamics of the LSS dominate important processes such as entrainment, mixing, momentum transport, and noise generation.

These findings motivated extensive research activities in the 1970s and 1980s in active flow control using excitation of instabilities. The experimental research focused nearly exclusively on the control of low-speed, low-Reynolds-number flows. This was due to the constraints imposed by then-available actuators: acoustic and mechanical actuators. As the flow speed and Reynolds number increase, so do the background noise, turbulence level, and instability frequencies, requiring excitation signals of much higher amplitude and frequency. These two opposing requirements impose a significant demand on the mechanical and acoustic actuators, which they have been historically unable to meet.

Although the foundation of our knowledge and understanding of instabilities and LSS is based on low-speed, low-Reynolds-number FSL, as detailed in this paper and summarized in this section, the nature of the IW and LSS and their evolution is quite similar over the wide range of flow speeds and Reynolds numbers more recently examined. Our recent work using localized arc filament plasma actuators in jets shows that FSL respond to the excitation over a wide range of variables that we have explored, including jet Mach number, convective Mach number, and Reynolds number up to 1.65, 1, and $1.65 \times 10^5$, respectively. However, the nature of LSS, shear-layer growth rate, and Mach wave generation all depend on the compressibility level. The experimental results presented in this paper and throughout the literature clearly demonstrate the similarity of instability processes and development of LSS in FSL, regardless of the Mach or Reynolds number.

Instability of FSL is discussed in Secs. II and III. A salient feature of an FSL is that the vorticity profile has a maximum (the velocity profile has an inflection point), and at sufficiently high Reynolds numbers, they are known to be unstable to perturbations over a wide range of frequencies. The most-amplified frequency scales with the average velocity across the shear layer and the shear-layer momentum thickness, $St_m = f_0 \theta / U_{ave} \sim 0.032$. This instability is called the Kelvin–Helmholtz instability. In a canonical FSL, the most influential parameters in determining the growth of disturbances and development of LSS are the normalized velocity difference across the FSL, the Reynolds number based on the initial momentum thickness, and the disturbance environment. Although various levels of analyses, from nondiverging to slowly diverging and from linear to nonlinear, have been used to explore the instability of FSL, all the evidence points to the fact that the growth of IW and their roll-up into LSS is quasi-linear and inviscid in nature at sufficiently high Reynolds numbers. Transition location depends on the Reynolds number and the level and nature of disturbances in a given facility. Additionally, although large-scale, coherent structures clearly develop over a wide range of Reynolds numbers, the level of small-scale structures embedded in the LSS increases with Reynolds number, reducing their coherency and two-dimensionality and making their identification more challenging. As the flow speed increases, compressibility effects begin altering the nature of the IW, LSS, and growth and development of the FSL. The convective Mach number has been used as a measure of compressibility. Significant reduction in the perturbation growth rate, turbulence level, mixing-layer growth, and increased three-dimensionality and lack of coherence of the LSS have been observed as $M_c$ is increased.

Growth and development of perturbations in a FSL, their roll-up into LSS, and their interaction are discussed in Secs. III and IV. In experimental work and applications, the naturally generated perturbations are generally broadband in both frequency and amplitude; however, instability analyses and carefully designed low-speed experiments using single-frequency perturbations have provided a wealth of information. The results indicate that when the excitation frequency is close to the most-amplified frequency $(1/2 f_s \leq f_e \leq f_s)$, the frequency of the shear-layer response is the same as the excitation frequency $(f_e = f_s)$, called the “lock-on control”, and the required perturbation amplitude for the excitation is quite small. In the lock-on control, the pairing of structures is discouraged (in low-speed, low-Reynolds-number flows). On the other hand, when the excitation frequency is much lower than the most-amplified frequency $(1/3 f_s < f_e < 1/2 f_s)$, the frequency of the structures jumps to the first excitation harmonic $2f_s$ closest to $f_e$, the excitation frequency becomes the first subharmonic of $f_s$, and the generated vortices go through multiple pairing processes. However, the required excitation amplitude for this case increases significantly. As discussed in Sec. VII.B, Mach 0.9 and Reynolds number of $6.2 \times 10^4$ are the highest Mach and Reynolds numbers at which merging of LSS in FSL has been observed experimentally.

The focus of the discussion in Secs. II–IV is on planar FSL. Free shear layers in other classes of canonical FSL, in which there is an additional length scale (e.g., free jets, cavity flows, and separated flows) are discussed in Sec. V. The second length scale in these flows is the jet nozzle exit diameter, the cavity length, and the length of the separation region, respectively. This secondary length scale establishes another dominant frequency in the flow, generally an order of magnitude below the most-amplified shear-layer frequency. This second frequency has different names in different flows/applications, it scales with the second length scale $(St_p = f_p U_i / D)$, and it is associated with the passage frequency of the LSS farther downstream in the shear layer. In the example flows listed previously, this second frequency is called the preferred-mode frequency, the Rossiter modes, and the shedding frequency, respectively. To limit the scope of the paper to a manageable level, Sec. V and the recent results presented in Sec. VII are limited to the discussion of axisymmetric free jets.

In an axisymmetric jet, the mixing layer has a ring shape. It begins similar to an FSL, but as the ring radius and thickness increase with streamwise location, the outer diameter increases, and the inner diameter decreases and eventually becomes zero. This occurs when the mixing layer closes on itself at the jet centerline. This location is called the end of the jet potential core. Though the jet potential core length changes dynamically with time, its time-averaged length scales primarily with the nozzle exit diameter and is on the order of $5D$ to $6D$, depending on the jet Reynolds number, perturbation environment, and Mach number, among other parameters. The initial shear-layer thickness is normally much smaller than the nozzle exit diameter. Therefore, the azimuthal curvature effect can typically be neglected, and the initial shear-layer development is quite similar to the development of planar FSL, and has been analyzed using linear stability theory. However, there are two major differences between the initial development of the IW in two-dimensional FSL and jets: first, the limited length of the potential core in jets, and second, the existence of azimuthal modes in axisymmetric jets. After the amplification and roll-up of the IW into LSS in planar FSL, the further development of the LSS heavily depends on the nature of the disturbances in the flow, whether naturally or artificially imposed. In low-speed and low-Reynolds-number flows, the structures may continue convecting downstream for a long time or quickly begin the process of pairing, depending on the disturbance environment. The amplification and roll-up of the IW into LSS in jets are initially similar to those in planar FSL, and the frequency of these structures is the most-amplified shear-layer frequency $f_e$. However, the passage frequency of the structures around the end of potential core is $f_p$, which is typically an order of magnitude smaller than $f_e$. Therefore, either multiple pairings must take place between the structure roll-up...
and the end of potential core or the shear layer must grow fast enough for $f_D$ to approach $f_p D$. The second difference between planar and axisymmetric FSL is the existence of azimuthal modes in the latter. Axisymmetric jets often display unstable azimuthal modes, in addition to the shear layer and jet column modes. The principal factors determining the existence and growth of azimuthal modes in the potential core region of jet FSL is the ratio of the nozzle exit diameter to the shear-layer momentum thickness $D/\theta$ and the disturbance environment. Linear stability analysis, experimental work, and modal analysis all show that, for a very thin shear layer (very large $D/\theta$), many azimuthal modes are unstable in the initial shear-layer region and that the number of unstable modes decreases as the shear-layer thickness increases. The research also shows that Reynolds number and Mach number (in addition to $D/\theta$ and disturbance environment) play a significant role in the selection of the dominant unstable mode.

Actuators are the backbone of any active flow control effort and have been reviewed extensively in recent years. Therefore, only thermal-based plasma actuators, especially localized arc filament plasma actuators (LAFPAs), which have been used to obtain the results in Sec. VII, have been discussed in detail in Sec. VI. They provide localized thermal perturbations with high amplitude and wide bandwidth for excitation of instabilities in high-speed and high-Reynolds-number flows. The frequency and phase of these actuators are independently controlled, allowing several to be used in tandem to excite not only the free shear-layer instability but also, for example, various azimuthal modes in axisymmetric free jets. The nature of the perturbations that these actuators generate and their excitation mechanism are also briefly discussed in Sec. VI. Thermal-based actuators for excitation of instabilities (including LAFPAs and NS-DBDs) have ushered in the most recent developments in flow control. The high-bandwidth, high-amplitude perturbations that these devices generate and their dynamic flexibility have not only opened the possibility of excitation of natural flow instabilities over practically all flow speeds and Reynolds numbers, especially previously inaccessible high-speed, high-Reynolds-number flows, but they have also enabled optimizing, dynamic feedback control in both these and other flows. The flow control community has now been exploring these exciting new areas for nearly two decades.

Section VII of the paper focuses on the most recent developments in understanding of the flow physics and control of high-speed free jets, using LAFPAs. The role of control in providing a better understanding of the underlying flow physics and the similarities between low-speed and high-speed FSL physics are highlighted. The results show that, at very low Strouhal numbers ($St_D < 0.1$), the excitation produces a compact waveform with a temporal persistence (i.e., wave period) much less than the excitation period. The reason for this phenomenon is the existence of higher harmonics of the actuation signal. Therefore, one of the harmonics of the actuation signal, which is closest to the most-amplified frequency of the jet shear layer, is amplified and rolls up into a structure. This response is referred to as the impulse response to the excitation. As the actuation Strouhal number increases to near the preferred-mode Strouhal number, the jet response matches the period of excitation. This is termed the periodic response of the jet. For excitation Strouhal numbers below the jet column Strouhal number ($St_D < 0.3$), the fundamental shape of the response is still largely unchanged from the impulse response, though at higher frequencies (e.g., $St_D = 0.50$), a distinct reduction in the fluctuation amplitude is apparent. However, it was found that this periodic response could be well approximated by a linear superposition of the impulse response, repeated at the higher excitation frequency with the proper phase shift. Thus, the jet responds to excitation over a large range of conditions explored in this work: Mach numbers, convective Mach numbers, and Reynolds numbers, up to 1.65, 1.0, and $1.65 \times 10^6$, respectively. Though the nature of LSS, shear-layer growth rate, and generation of Mach waves all depend on the jet Mach number and compressibility level, these results clearly demonstrate the overall similarity of instability processes and development of LSS, regardless of the FSL Reynolds number and Mach number. Similarities were also observed in the response of the jet to excitation of azimuthal modes near the jet preferred-mode frequency. These results highlight the fundamental nature of the instabilities present in jet FSL and their relative insensitivity to jet Mach and Reynolds number.

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References


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